AU/ACSC/0229/97-03

CHAOS THEORY FOR THE PRACTICAL MILITARY MIND

A Research Paper

Presented To

The Research Department

Air Command and Staff College

In Partial Fulfillment of the Graduation Requirements of ACSC

by

Maj. Susan E. Durham, Ph.D.

March 1997

Disclaimer

The views expressed in this academic research paper are those of the author(s) and do not reflect the official policy or position of the US government or the Department of Defense.

Contents

Illustrations

Page

Acknowledgments

I wish to thank my Faculty Research Advisor at Air Command and Staff College, Lt Col Thomas Kelso, Ph.D., for letting me do the impractical: letting me switch research topics several weeks into cycle, simply because I'd become intrigued with the subject of Chaos. His advice and encouragement were indispensable. I hope this product is worthy of his faith in my abilities.

I am grateful to Lt Col (S) Glen James, Ph.D. whose prior work is the basis of this paper, and whose graciousness in excepting me into the field was most encouraging.

Most importantly, I would like to thank my husband, not only for taking over all my chores while I labored over this research, but who, for the past twenty years, has put up with my frenzied search for answers to which I often don't know the questions. He is my loving partner, my steadfast ally, my hero, and my greatest joy.

Last, I must mention that I am grateful for my dog, Marlow, who knew when I needed a break, even when I didn't know. That insistent head on my keyboard and those soft brown eyes begging for a game of tug-of-war gave me much needed stress relief. Thank God for big dogs.

Abstract

The military professional is a practically-minded individual. This is not, stereotypes aside, the result of an inflexible, unimaginative nature, but comes from pursuing a profession that emphasizes mission accomplishment above all else. What sane nation would want less of its protectors?

This paper is written with such a practical mindset, and begins with a definition of the most pertinent aspects of Chaos Theory for military applications. This is kept at a conceptual level for the benefit of the novice looking to understand the 'big picture' before pursuing the topic further, and for those individuals who do not *need* to work at a more mathematical level. Examples of Chaotic systems of military interest are given.

This work also addresses some of the difficulties in applying this mathematical theory metaphorically, and to social situations. For, although it is still being developed, Chaos Theory is being exploited by military strategists, economists, political analysts, and others with results that range from pragmatic and useful, to fanciful nonsense. The military professional could benefit from understanding some of the pitfalls of potential misapplication of Chaos Theory.

Last, this paper provides an open-ended discussion of *how* to apply Chaos Theory, by stepping the reader through the process of evaluating a system that is not strictly physical, for the potential applicability of Chaos Theory.

Chapter 1

Chaos With a Capital ‘C’

And now for something completely different.

—John Cleese
Monty Python’s Flying Circus

Everyone knows what ‘chaos’ is. It’s a department store on Christmas Eve, a birthday party for two-year-olds where someone lets a puppy loose, and the state your files are in when the boss announces an I.G. inspection is coming in six weeks. It is, according to the dictionary, disorder and confusion. But people often interpret that to mean chaos is also random, unpredictable, and uncontrollable. For systems involving human behavior as a driving force like those above, this is not a bad approximation. But in the case of real, physical systems that are chaotic, that assumption is wrong. Chaos in physical systems simply means disorder—wild fluctuations in output—arising from non-random causes. At first, this might seem like a trivial matter of semantics used by persnickety math geeks to ‘talk down’ to the public. It is not. The difference is significant because it concerns matters of predictability and control: very important issues in a mission-oriented profession like the military. In fact, one of the military’s most erudite Chaologists, Glenn James, in his excellent tutorial Chaos: The Essentials for Military Applications, specifically defines Chaos with a capital ‘C,’ in order to highlight the difference between true mathematical Chaos that occurs in real, physical systems, and the more vernacular, 'societal' confusion.[1] We'll follow his lead.

Further, since this paper is aimed primarily at the military professionals—active duty and civilians—who don't need to get into the mathematical 'weeds' to get their job done, yet needs to understand the effects of Chaos, we'll keep the level conceptual and as non-mathematical as practical.

While we will develop definitions throughout this paper, key concepts that are critical to understanding why Chaos Theory is relevant to the military are:

1. Chaos is *not* randomness and it does not arise from the same stochastic forces that cause random behavior;
2. Instead, Chaos arises from the same completely knowable conditions that give rise to ordered, thoroughly predictable behavior, *despite* the disorder of Chaos itself;
3. As a result, Chaotic systems can often be mistaken for random systems, and the potential for well-behaved systems to become Chaotic is often not realized;
4. There is an underlying structure to Chaotic systems that sometimes allow us to make *predictions* about its long-term trend, and *very* short-term behavior;
5. Some Chaotic systems can be driven in or out of Chaos; that is, Chaos can occasionally be *controlled*.

These are bold assertions, that may even seem counter-intuitive at first. But they are important for three reasons. First, because there are countless, real, physical systems upon which military lives and missions depend that are Chaotic. We will look at examples shortly. Second, since random systems can't be controlled and are unpredictable, when we mistake Chaotic behavior for random behavior we risk losing even the limited ability to make predictions of the system. This is tantamount to abdicating control. Third, when we don't recognize the potential in well-behaved systems to deteriorate suddenly into Chaotic behavior, we also risk losing control.

But, what kind of military systems are Chaotic, or could become Chaotic? Turbulence, for example, is Chaotic. Anything affected by turbulence, like aircraft wings, paratroopers making a jump, water lines, lasers propagating through the atmosphere, or imaging to and from space, and so on, are subject to Chaotic forces. The weather is a Chaotic system, and anyone who flies, sails, or treks across deserts and through forests is affected by that. Transient effects like wind gusts or waves can cause a system to go Chaotic; this can, and has, resulted in capsizing of ships thought to be stable to these influences. Electrical circuits can be driven in and out of Chaos, as many of our communications specialists already know. Many systems of the human body are known to be, or suspected of being Chaotic, like brain activity and the rhythm of the beating heart. And as more is becoming known about Chaos, a myriad of applications important to the military are sprouting up. Data compression, important for satellite links, might be accomplished by applying Chaotic analysis to transmission techniques. New ways of making and breaking codes are being worked on. Imagine, if you will, our enemies having the ability to transmit code that is mistaken by us for random noise! That kind of vulnerability we just can't afford. In short, Chaos can show up in places where one might least suspect. This has the potential to cost lives, risk missions, and give our enemies an operational advantage. Clearly, Chaos is a subject worth investigating.

On the other hand turbulence, weather, and other such systems have been around forever, and the world has not imploded because these systems weren't recognized as Chaotic. Why the hullabaloo now? Partly because the mathematical aspects of Chaos have only been recognized in the last few decades. Previously, order and randomness were seen as the only two world-views available. If something behaved in a disordered fashion, it was assumed that either random forces were at hand, or not everything was known about the system. But Chaos is a *fundamentally different way of viewing reality*; it is a type of behavior that has characteristics in common with both order and randomness, but is not either. This realization, however, didn't come about until about 1960, with the ground-breaking work of meteorologist Edward Lorenz.. Thus, part of the reason Chaos Theory is only now becoming known, is that it is still being developed in a mathematical sense—and still has a long way to go.

But the sudden interest in Chaos Theory is also due, in part, to the recent popularization of the subject by such authors as James Gleick in his enlightening book *Chaos: Making a New Science*. (It can be argued that Gleick has done for Chaos Theory what Carl Sagan did for astronomy: educated and intrigued the masses in a way that no collection of pedantic equation-strewn monographs ever could.) This is good and bad. It's good because it makes Chaos accessible to the vast, creative forces of collective human intelligence. Who knows what can ultimately come of that? It's bad because Chaos Theory is a mathematical theory, and like most tools of 'hard' science doesn't readily lend itself to 'soft' science application. However, that hasn't stopped economists, political scientists, sociologists, anthropologists and the like from trying to apply the mathematical concepts of Chaos to 'societal' applications, often with results that range from practical and useful to fanciful nonsense. This is important to the military professional because Chaos Theory is becoming an increasingly popular topic in the fields of strategy, economics and politics. The military strategist, in particular, is likely to see Chaos applied more frequently to military issues, both metaphorically—much in the same way Clausewitz applied Newtonian concepts like 'centers of gravity'—and as an analysis tool of non-mathematical situations of warfare and conflict. The military professional could benefit from understanding some of the pitfalls of potential misapplication of Chaos Theory. Accordingly, this paper also discusses the risks in applying the ‘hard’ science tool of Chaos Theory to ‘soft’ science military issues.

Last, this paper also presents a brief exercise in how to ‘think Chaotically’, by walking the reader through the kinds of questions to ask when deciding if Chaos Theory can be applied to systems that are not strictly of the ‘engineering’ sort.

But before stepping over the precipice into Chaos, so to speak, the author would like to emphasize that the scope of this work is quite narrow; it focuses on those aspects of Chaos which specifically deal with issues of predictability. For the reader interested in a fuller, more in-depth treatment of Chaos Theory, the author *highly recommends* the fine tutorial on Chaos theory by Glenn James, mentioned above. Specifically written with the military in mind, this tutorial is aimed at the average non-scientist and is very readable, as well as insightful. This work capitalizes heavily on James’ paper, as well as the works of Gleick and DuBlois, also recommend to the reader who wants to delve deeper into the ‘something completely different’ of Chaos Theory.[2]

Notes

[1] James, Glenn. *Chaos Theory: The Essentials for Military Applications,* Newport, R.I., Naval War College, 1995, 3.

[2] DeBlois, Bruce. *Deterministic Philosophical Assumptions in the Application of Chaos Theory to Social Events*, Maxwell AFB, School of Advanced Airpower Studies, not published and Gleick, James. *Chaos: Making a New Science,* New York, Penguin Books, 1988.

Chapter 2

Piercing The 'Fog' Of Chaos

> *…all action takes place, so to speak, in a kind of twilight, which, like fog or moonlight, often tends to make things seem grotesque and larger than they really are.*
>
> —Clausewitz

From the generalities presented so far, Chaos must certainly seem insidious: a state of disorder, masquerading as randomness, yet arising from conditions that—post-Newtonian common sense says—should yield complete predictability. But what does this mean? It seems to imply that Chaos is as nebulous and intractable as Clausewitz's 'fog of war.'[1] But any commander worth his or her rank knows that piercing the fog of war lies in sorting out what is, isn't, can, and simply *can't* be known in battle; acting on what is knowable; and *dealing* with unavoidable unknowables. In that vein, this chapter is intended to help the reader 'pierce the fog of Chaos.' By burning away the nebulous and erroneous notions most people have about Chaos Theory—in essence, seeing what is, isn't, can, and can't be known about Chaos in physical system—the author hopes to help the reader learn to recognize Chaos in military systems, so as to control it when they can control it, and deal with it when they can't.

Birds, Bass, Beetles, and Buffalo

To understand where Chaos comes from, let's begin by looking at a well-behaved system that becomes Chaotic. Such a system can be found in population biology: the study of the life and death cycles of birds, bass, beetles, and buffalo, to name a few.

Biologists are very interested in predicting fluctuations in wildlife populations. This may seem like an enormously complex task, but in actuality, one simple equation, called the *logistics equation,* approximates observed behavior with great accuracy. (Military professionals dealing with ecological issues such as base-closure clean-up, or biological issues such as biological warfare are probably already familiar with this equation.) We won't explore the equation itself in the main text; it's explained more fully in Appendix A. At a conceptual level, the reader only needs to understand that the equation allows us to predict variation in population based on *only two factors:*

1. the average number of offspring per adult (a constant), and
2. the initial population.

This is an iterative equation, meaning that having calculated one year's population, that value is input back into the equation to predict the next year's, and so on. A key aspect of the equation, however, is a feedback factor that depends only on the population value as it changes year to year. When the population becomes too big for the local ecosystem to support it, the feedback factor dampens the population. When it is smaller, the feedback 'encourages' higher future populations. What is most important about this feedback factor is that it introduces nonlinearity into the system. This nonlinearity is not so different from what people mean when they describe a hot-tempered individual as going 'nonlinear' if anyone even mildly disagrees with him or her, meaning that person's response was out of proportion with the situation. Similarly, by definition, nonlinearity in a system means that the output is not directly or inversely proportional to the input. Linear equations contain only addition, subtraction, multiplication or division by constants. Nonlinear operations involve exponents, trigonometric functions, and logarithms. One of the fundamental truths about Chaos is all Chaotic systems are nonlinear, but not all nonlinear systems are Chaotic. Further, as we'll see later in this paper, linear systems are never Chaotic. Many Chaotic systems become so because they are subject to this type of nonlinear feedback, which the system can't 'compensate' for. The result are wild fluctuations characteristic of Chaos. We can see this in 'watching' our population biology example go from stable, to Chaotic.

Consider how the population of bass changes as described by the logistics equation. Assume a near-zero population initially (near-extinction) and an average 2.0 offspring per adult. Figure 1 shows the results. Notice that the population of bass rises to a constant value, year after year remaining the same.[2] In this case, although nonlinear feedback exists in the system, its effects are negligible in cases of low birth rates (see Appendix A). An individual running a 'farm-pond' would be happy with these results.

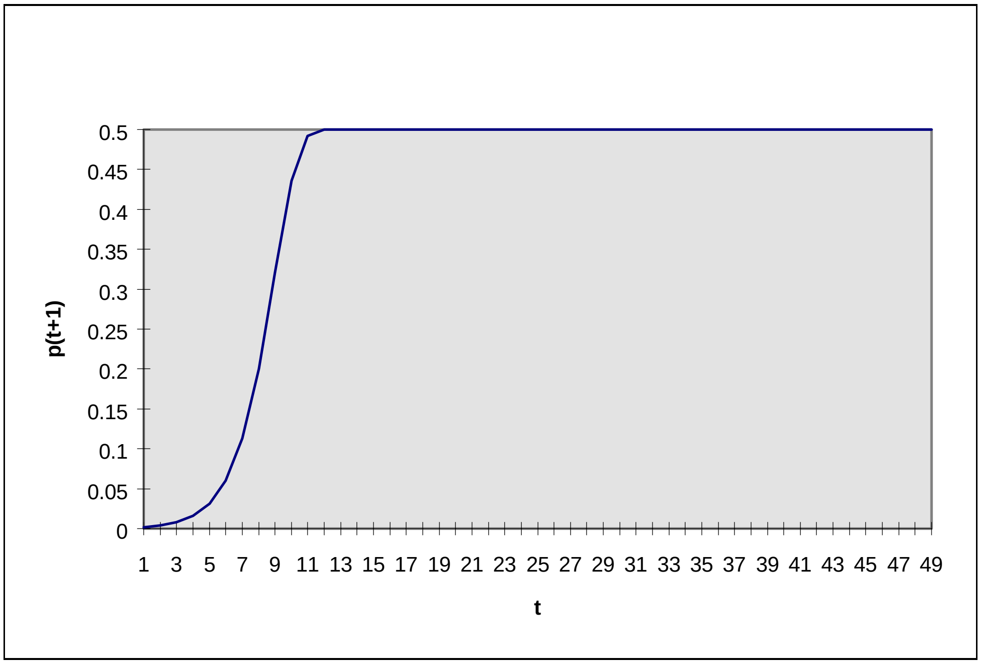

Figure 1. Steady State Solution of Logistics Equation: λ = 2.0

But something curious happens when the number of offspring increases to more than 3 per adult. Figure 2 shows the case of an average of 3.4 offspring. Here, the population attains a periodic pattern; steady high one year, low the next, and so on. What's happening is that the population rises so quickly that it initially overshoots a stable solution, then feedback causes a depletion in the next year. But the feedback encourages too much depletion, so the system again compensates, causing another boom and so on and so on. Such a sudden change in the character of the output is known as a bifurcation, another term that frequently appears when defining Chaos (and one that many authors tend to misuse). A bifurcation simply means a drastic change in the pattern, or the quantitative state of a system. The type of bifurcation shown in Figure 2 is what is called period doubling, or period-2. What's very important to realize here is that the only difference between the steady state solution shown in Figure 1, and the period doubling that occurs in Figure 2, is that we've changed a constant, nothing else. That is, year after year after year we assume the average bass can produce 3.4 offspring per adult. This increase in the constant, increases the effect the nonlinear feedback has on the system.

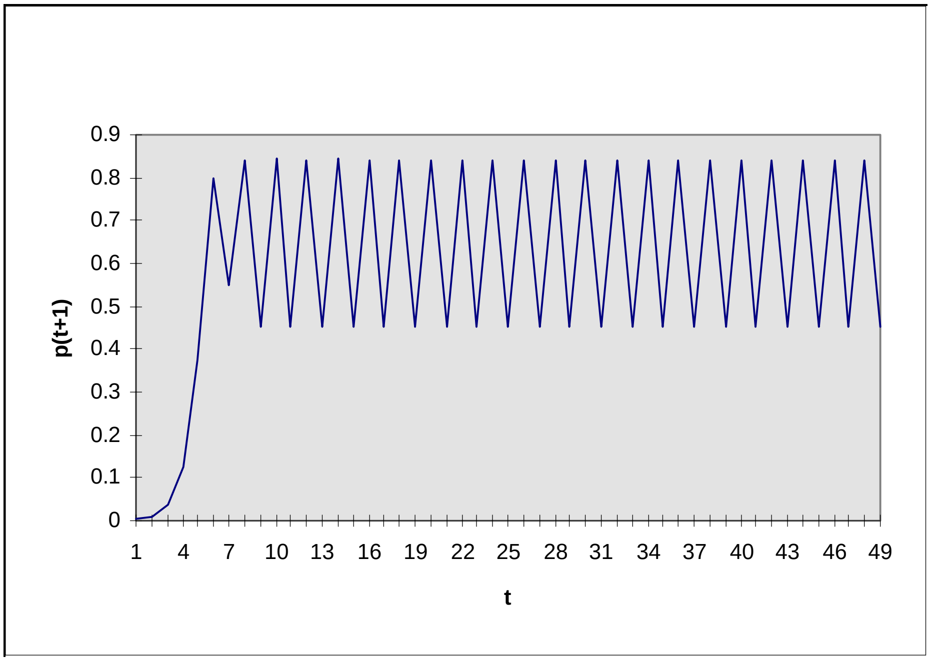

Figure 2. Period-2: $\lambda = 3.4$

Now if we have an even higher offspring rate (about 3.6 offspring per adult) the nonlinear feedback induces even more dynamical change. The period doubles again to what is called period-4, as shown in Figure 3. Notice that although we have fluctuating values, like in Figure 2, the fluctuations are steady, repeatable, completely predictable. Once we know what the first four years bring, and as long as we don't introduce something new into the system, we can predict the bass population forever! This is not mathematical trickery, or naive model-making. Real populations behave this way.

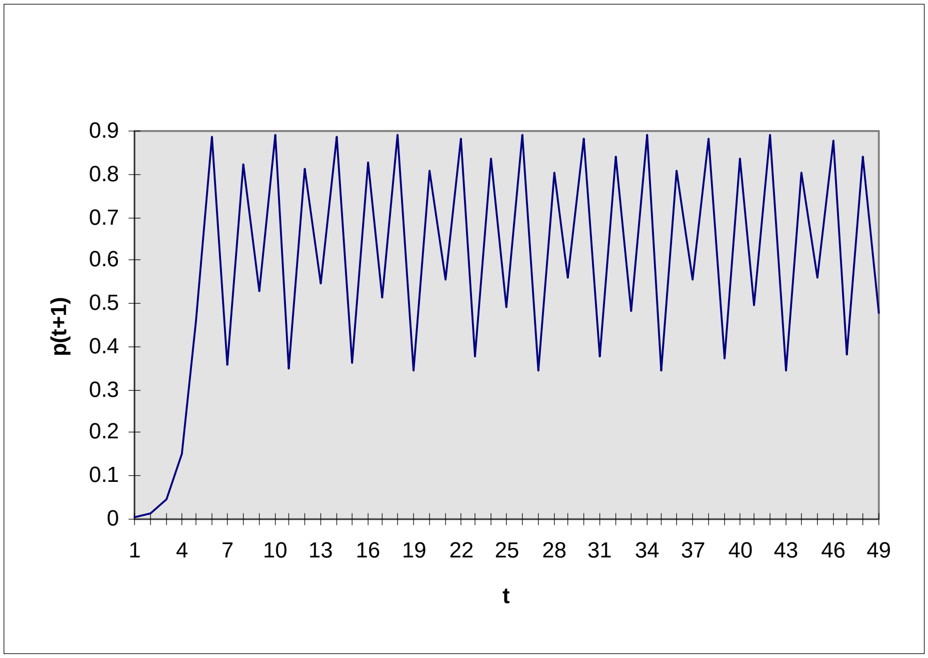

Figure 3. Period-4: λ = 3.5699

What we've seen in the previous examples of bifurcation is simply the effects of quite *manageable feedback*. But as anyone who has ever set up their own audio system knows, feedback can become quite unmanageable. When the feedback is too large, its nonlinear effects dominate the systems behavior. This is exactly what happens if we change the birthrate in the logistics equation to 4.0 (a mere 0.4 difference from our last example). The bass population begins to fluctuate wildly as shown in Figure 4. Suddenly we have Chaos! It is important to understand that these sudden wild fluctuations have arisen from the same completely known types of conditions that produced the steady state, and well-behaved periodicity seen in the previous examples. Why this happened in this example is a matter of feedback. *Feedback is a major factor in driving many systems into Chaos.* The nonlinearities which are manifest in feedback to the system, *already exist* and small changes in the physical conditions of a system can mean the difference between thoroughly characterizable systems and Chaotic systems.

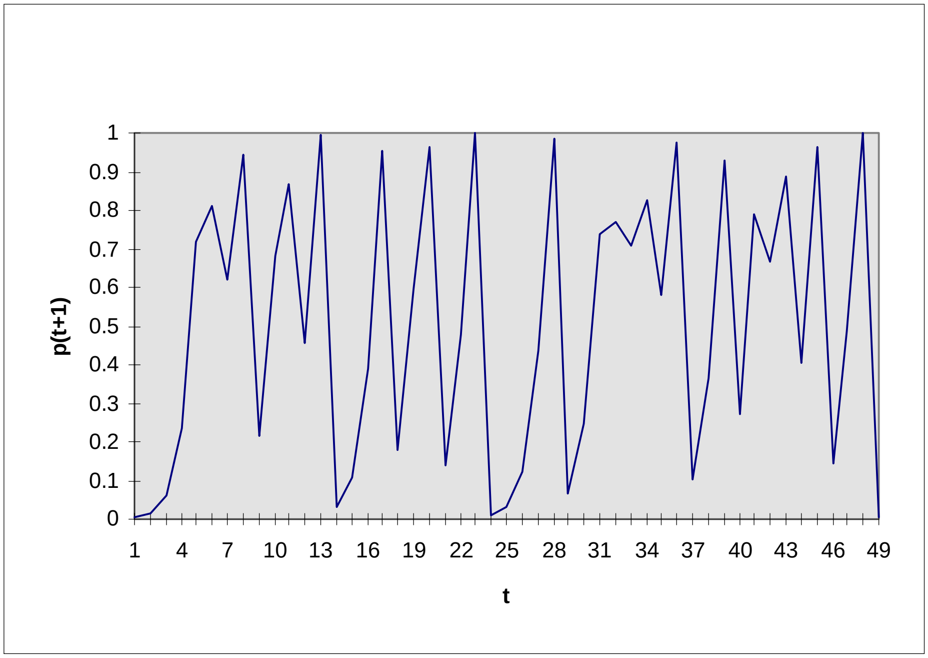

Figure 4. Chaos "sets in": λ = 4.0

By the way, looking at Figure 4, we can see why Chaos is often mistaken for randomness; it certainly *looks* random doesn't it? We'll discuss more on that issue later.

A New World View?

The example above was first seen as contrary to the Newtonian physics ideal of *determinism*. Determinism means that the future of any system can be *precisely* known if enough is known about the constituents of the system and the conditions effecting it: that is, what the system contains, what forces are acting on the system and how those forces change with time. Before Chaos Theory, scientists thought deterministic conditions *always* produced completely predictable behavior and that the only 'options' available to a system was:

1. total predictability based on deterministic, characterizable, conditions, or
2. disorder, based on random, stochastic processes. DeBlois neatly summarizes this Pre-Chaos World View as seen in Figure 5.

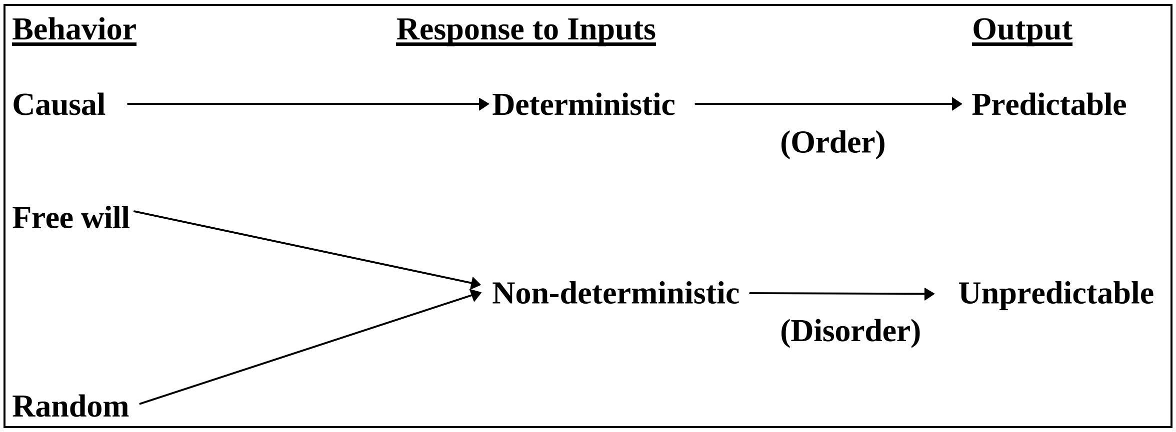

Figure 5. Pre-Chaos World View

But the logistics equation shows that this isn't true. In the example above, all the inputs to the system—the average birth rate and initial population—were completely known. The results from that point on were simply a matter of iterating on these values. There were *no* random inputs! We knew exactly what 'went into' the system, why couldn't we know exactly what 'came out?' Common sense would seem to dictate that the results should have been completely predictable. Yet, four solutions to the exact same equation, starting from the exact same initial population, differing only by a constant, yield results that vary from steady state to Chaos. This result is one of the most significant concerning Chaos. In fact, a second 'truth' of Chaos is that *Chaos results from completely known, deterministic, conditions.* Chaos is *not* caused by random events and Chaotic systems *do not* behave randomly, as we shall see.

Thus, we can argue that Chaos is a fundamentally new way of viewing reality as DeBlois, again, succinctly illustrates in Figure 6. We no longer can say that reality is *either* random or completely predictable. Why this is important to the military professional—and why we can continue to label Chaos as insidious—is that Chaos frequently 'sets-in' to systems that have only *minor differences in the physical conditions, or parameters,* from completely predictable systems.

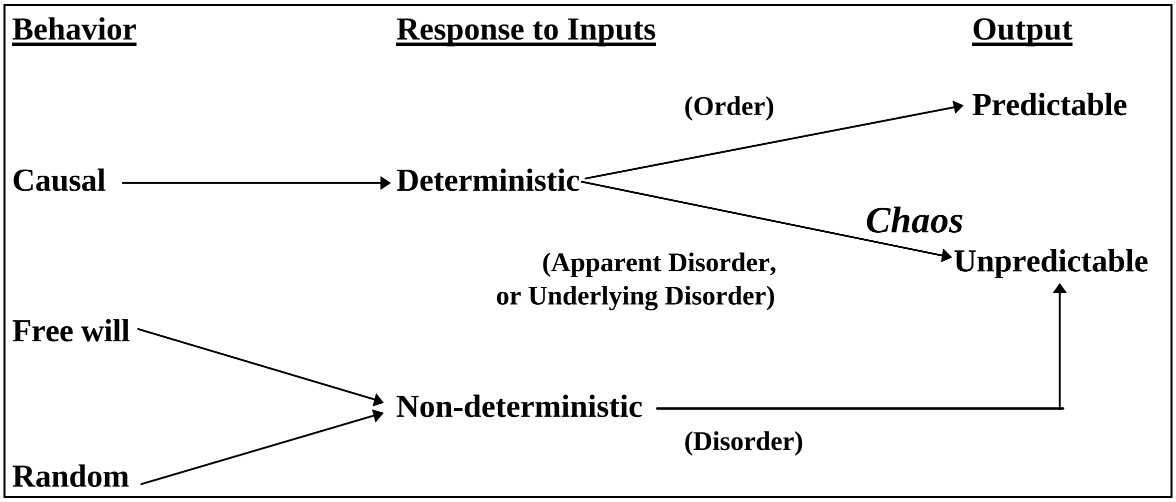

Figure 6. Generalized Post-Chaos World View

These parameters are constants throughout the evolution of the system (e.g., they don't change with time).[3] These are often known as *control parameters* and they are good news for the military, or for anyone wanting to drive a system into Chaos, or prevent Chaos from occurring. For example, in the bass population system, the control parameter was offspring per adult. Modify that, and the system won't become Chaotic, or can be 'pulled-back' from Chaos. However, knowing how to find these control parameters requires more background on the nature of how Chaotic systems behave, as will be explored in the next sections.

Butterflies and Hurricanes

Thus far we've developed two of the basic truths of Chaos: all Chaotic systems are nonlinear, and all Chaotic systems are deterministic. A third basic truth that differentiates Chaotic systems from all other systems is that *all Chaotic systems are sensitive to initial* *conditions* (SIC). To understand what this means, it helps to take a short time-trip backward to the initial 'discovery' of Chaos, as we shall now do.

The development of Chaos Theory was largely serendipitous and mostly unexpected. It wasn't that Chaos hadn't been observed. For centuries experimentalists and theorists alike knew of stable, periodic systems suddenly deteriorating into disordered, wildly fluctuating, behavior. But this was often blamed on poor data collection techniques, or incomplete knowledge of the system. After all, since Newton, the catechism of non-random physics was that of determinism, as we've discussed above. This world view got a severe upset when Edward Lorenz, a research meteorologist from the Massachusetts Institute of Technology, was attempting to computer-model weather.

He was doing this in 1960, when computers were still masses of wiring and tubes, and even punch cards were a luxury. Nonetheless, he was making good, if interminably slow progress, having reduced the basic forces that drive weather (pressure, temperature and wind speed) into relational equations. His computerized weather behaved much as he expected, generating highs, lows, jet streams, and seasons. But he noticed that a sort of "orderly disorder"[4] cropped up in the results. He was certainly seeing long-term trends—days got warmer in summer, cooler in winter, gradations in temperature signaled storms—all just as expected. Yet, within those general trends, the actual pattern was never quite the same; the day to day weather predictions differed greatly, even when his initial starting points (initial conditions) seemed almost exactly the same. Puzzled, Lorenz investigated the patterns more closely. One day, he made the fateful decision to study one particular "run" in more detail. Rather than starting the program from the beginning—a tedious task in the face of the unsophisticated computers of the time—he started from the middle, inputting the intermediate values from a previous run he had in hand. To his great surprise even though he was inputting the exact values as they read on the printout, he saw that the pattern that resulted from this run was markedly different from the previous one. Eventually, he realized that the intermediate values he had input from the printout had been truncated, but only by a very small amount. Yet, what a large difference this made! Although his new 'run' and the old 'run' displayed patterns that had same relative degree of complexity and the same overall "trend," (i.e., the 'high' and 'lows' were of the same order) they were undoubtedly, and very unexpectedly, vastly different. What Edward Lorenz had inadvertently stumbled onto was the phenomenon of *sensitivity to initial conditions* or SIC. Figure 7 shows an example of SIC.

Here we see two solutions to the exact same equation. If the solution shown with a lighter line looks familiar, that's because it is the same as Figure 4, the Chaotic solution to the logistics equation. The difference in the two solutions is *only* in the initial value of the population. In the light line the initial value of the population was 0.001. In the dark line it was set to 0.000999, a difference of only one part in one million. But look at the difference in patterns. After only ten time steps, the two lines, which perfectly coincide in the early years, diverge rapidly. By step 34, one equation is predicting a high while the other is predicting a low. In short, *the slightest variation during the time-evolution of the system result in vastly different outcomes.* Once again, *every* Chaotic system is sensitive to initial conditions. This sensitivity is a hallmark of Chaos.

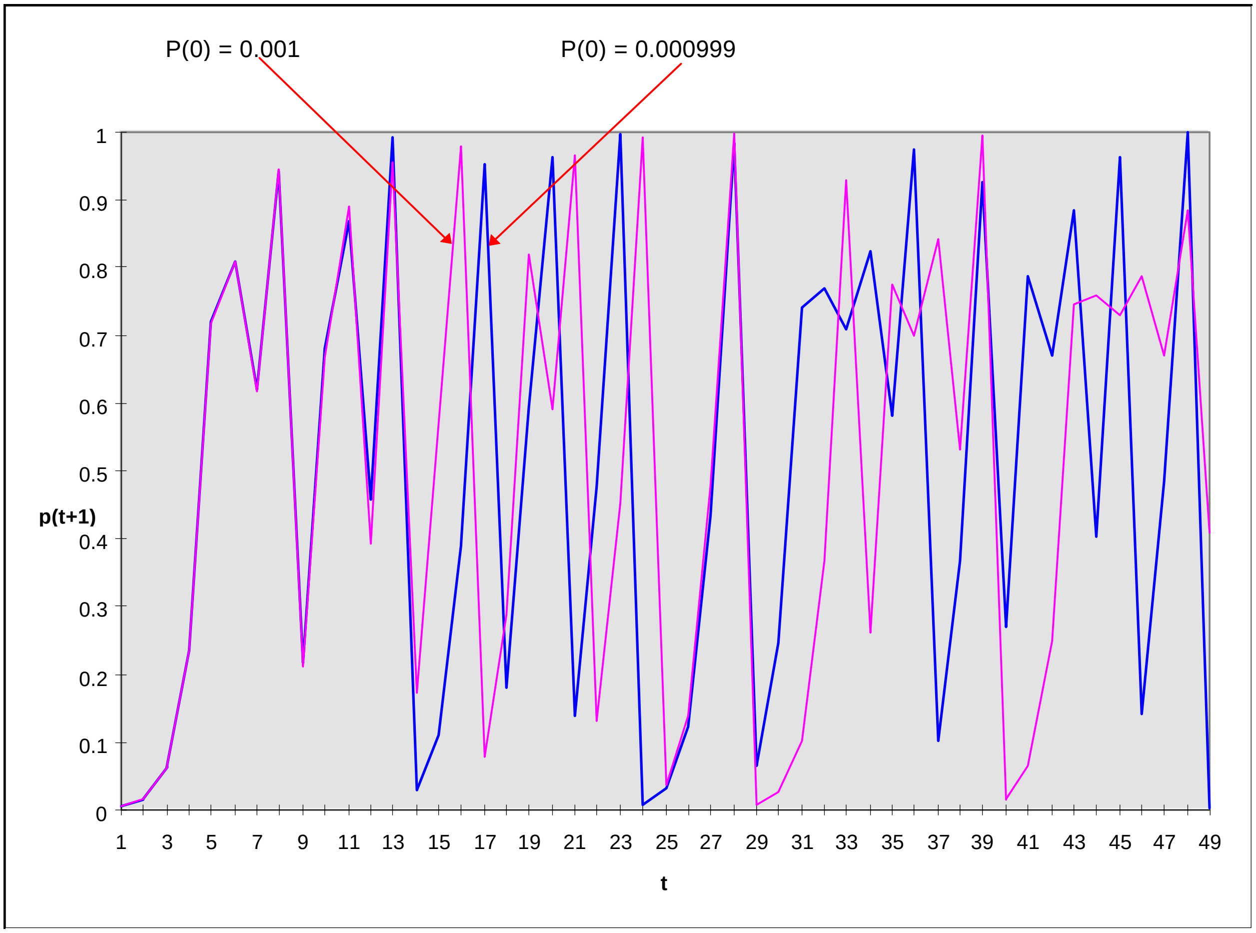

Figure 7. Sensitivity to Initial Conditions

For the sake of completeness, it should be noted that this sensitivity to initial condition is often referred to as the 'Butterfly Effect.' This simplification of Chaos Theory is often misused. The popularized notion of the Butterfly Effect goes something like "if a butterfly flaps its wings in Philadelphia, it can cause a hurricane in Japan." But this notion is wrong by one important word: 'cause.' The butterfly doesn't cause the hurricane. The system must *already* have enough energy in it to produce a hurricane. The presence of the butterfly merely disturbs the already Chaotic system, which is extremely sensitive to the smallest of changes, effectively 'sending the system' off in a different direction than if the butterfly wasn't there. This makes it impossible to *predict* if the hurricane will occur, just like changing the initial population of bass in our logistics equations by one part in one millionth made it impossible to predict in which years the population would 'boom' and in which years it would 'bust.'

This capricious nature of Chaotic systems seems to make the issue of prediction concerning Chaotic phenomenon seem hopeless. But it isn't—completely—hopeless. Thus far, the basic truths we've discovered about Chaos lead us to the conclusion that despite the *deterministic* origins of Chaos, its *nonlinear* nature (often manifest in system feedback), and its *sensitivity to initial conditions* results in extremely dynamic, disorderly results. Thus, it is a fourth major truth about Chaotic systems that *they are unpredictable except in terms of long-term trends and occasionally in the very short term.* In the next sections we'll see what is meant by these exceptions, learning under what circumstances we *can* make predictions about Chaotic systems, which lead us closer to controlling these systems.

Other "Fashionable" Concepts

In the highly entertaining—albeit highly fanciful movie—*Jurassic Park*, actor Jeff Goldblum plays a self-proclaimed Chaotician. In discussing the viability of the prehistoric theme park, Goldblum's character tosses out the terms phase-space, strange attractors, and more, in such an off-handed manner that the park's creator surmises Chaos Theory is just "a load of fashionable number crunching." Such a response is completely reasonable. The terms phase-space and strange attractors certainly appear daunting to non-mathematicians, causing many peoples eyes to glaze over with disinterest. But these are *not* difficult notions at the conceptual level no matter how 'high-faluting'—as the author's grandfather would say—they sound. And they are necessary to understanding what Chaos is. Thus, in this section we will examine the nature of phase-space, and then of attractors, in periodic systems. Subsequently, we'll extrapolate these concepts to define them in Chaotic and random systems. This will allow us to use phase-space to reveal the *hidden structure* in Chaotic systems, hence allowing us to make limited predictions about Chaotic behavior.

At the most basic level, phase-space is simply a way of graphically representing the way a system behaves. Take for example a simple, frictionless pendulum. This is a dynamical system; in other words a system that changes with time. One way to describe this system is to graph how the bob position changes from moment to moment. Consider Figure 8. If we define the position of the pendulum as zero when the bob is hanging straight down, then left of center is negative, and right of center is positive. As time progresses, the bob swings to one side then the other. Graphically this looks like a sine wave, as shown. Most people are familiar with the sine wave as one form of *periodic* behavior, so there's no surprise here.

But a more complete way to describe the system is to examine how the time- and space-dependent properties, or *variables,* of the system evolve. The variables gives us complete information about the condition, or *state,* of the pendulum *at a point in time*. Graphing the way the variables change *with respect to each other* is called representing the system in *phase-space*. In the case of the pendulum the variables are position and velocity. Let's now look at how to graph the evolution of these variables in phase-space. Consider Figure 9.[5] In the upper left of the figure we show the pendulum at its left-most position (remember, left-of-center is negative). Having traveled as 'high' as the energy in the system allows, the bob 'stops'; that is, its speed and velocity are zero. At that instant in time, the state of the system can be completely defined as a point in phase-space, this point has negative position and zero velocity. This point in phase-space is shown in the upper-right figure of Figure 9.

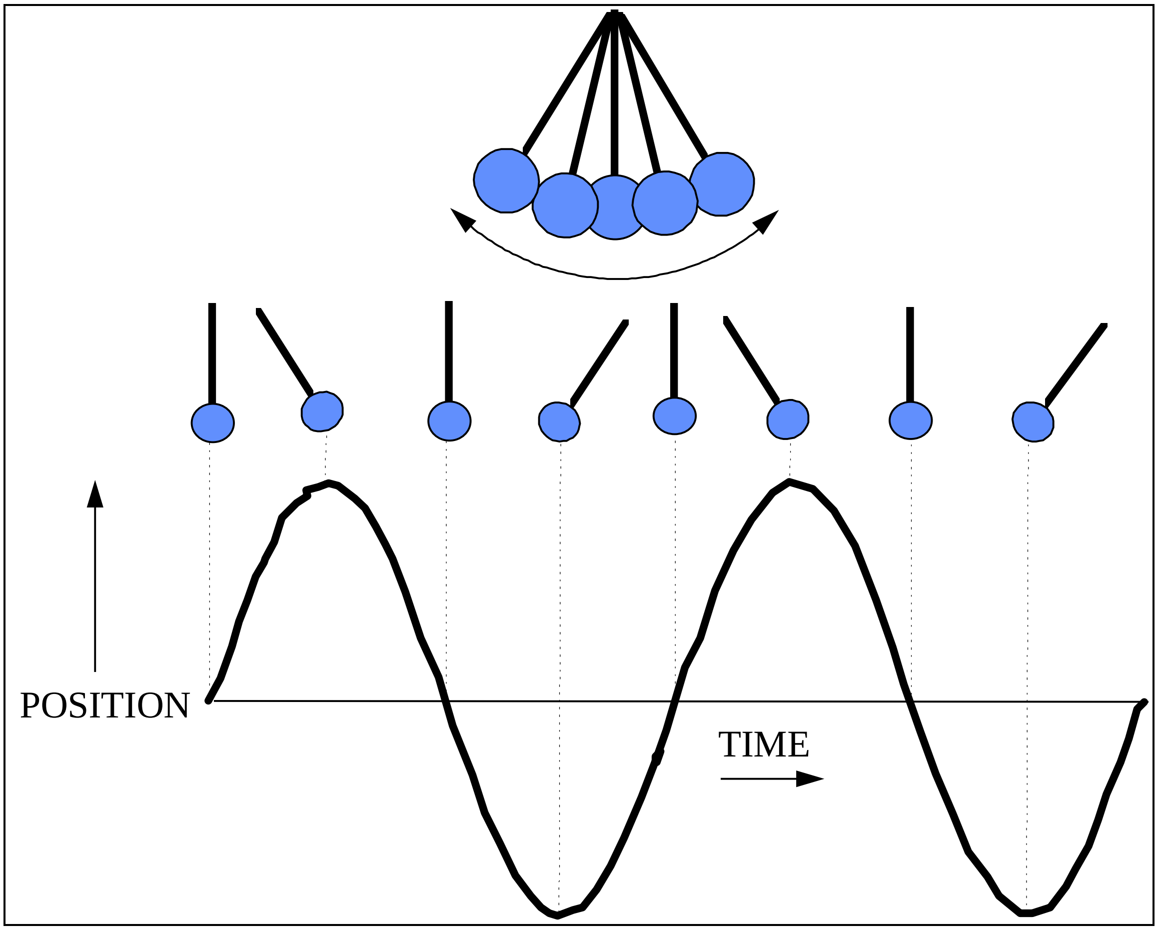

Figure 8. The Simple, Frictionless Pendulum

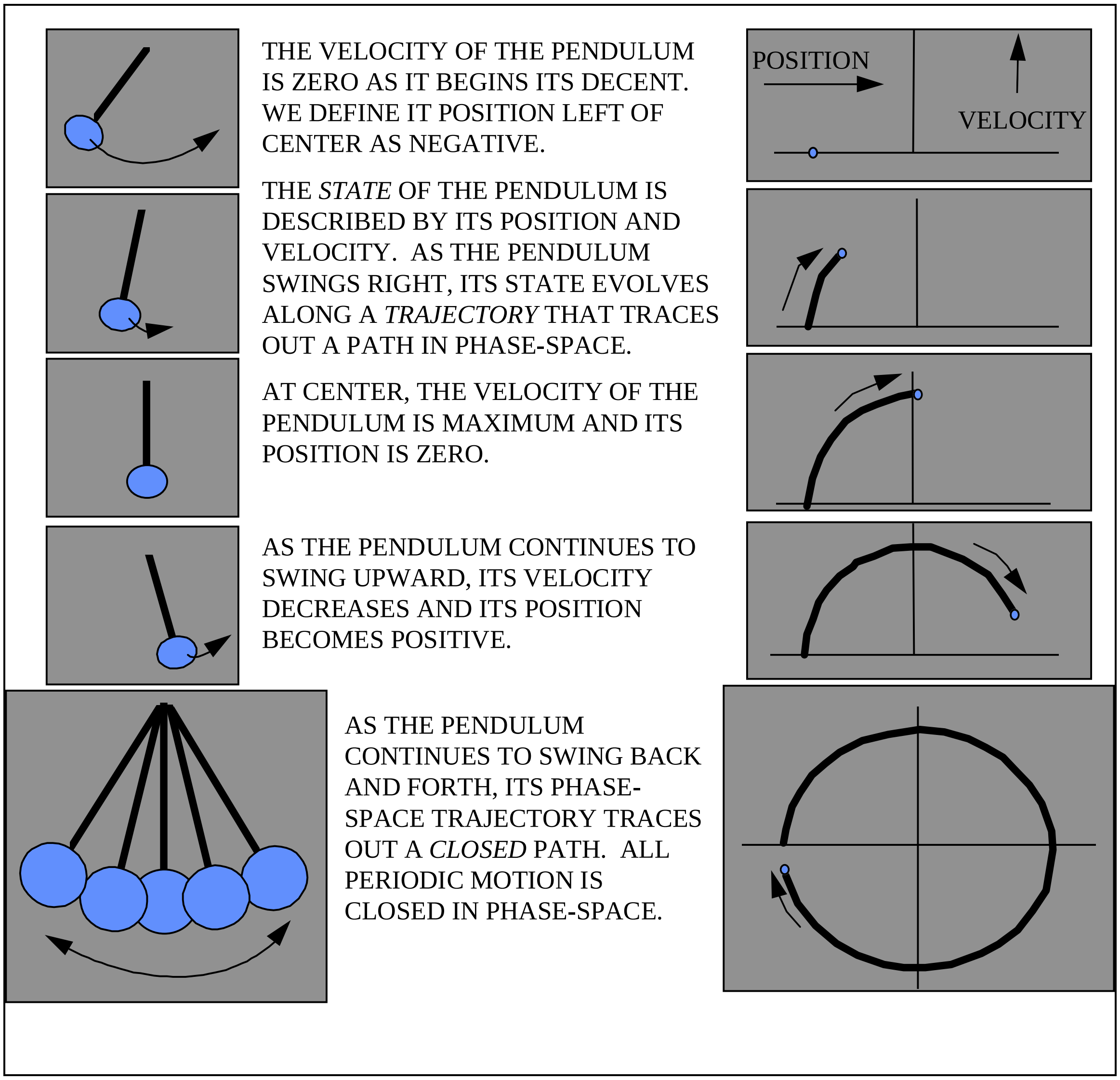

Figure 9. The Simple, Frictionless Pendulum in Phase-Space

As the bob falls, it speeds up. Since it is going in a positive direction, its velocity (speed *and* position) increases. The state of the system evolves along a *trajectory* in phase-space as shown. This trajectory traces out an *orbit* in phase-space. The actual state of the system is still a point. When the pendulum reaches center, its position is zero (by definition) and its velocity is maximum. As it rises to the right of center, the position continues to become more positive, but the velocity decreases. The system evolves in phase-space as the pendulum swings, until it returns to *exactly* the same spot where it began. When this happens, its trajectory in phase-space closes into an ellipse. All periodic behavior shows up as closed loops in phase-space.

There is one more important aspect of the pendulum in phase-space we need to discuss before moving onto Chaos and randomness; that of *attractors*. In the case of the frictionless pendulum, the attractor is the *focus* of the phase-space orbit a virtual point about which the trajectory orbits. This is conceptually similar to the Sun being the focus of the orbit of the Earth. But in our example, we assumed a frictionless pendulum. In reality, friction is always present, and unless we drive the pendulum's motion with some external force, it will eventually succumb to friction and come to a complete rest. In phase-space, its trajectory would not close, but would spiral inward until the bob reached a steady state of zero position and zero velocity, as shown in Figure 10. In this case the attractor literally attracts or 'draws' the trajectory in phase-space to it. One can think of the attractor of a periodic system as that point that the system would eventually collapse to in phase-space if no external forces act upon it.

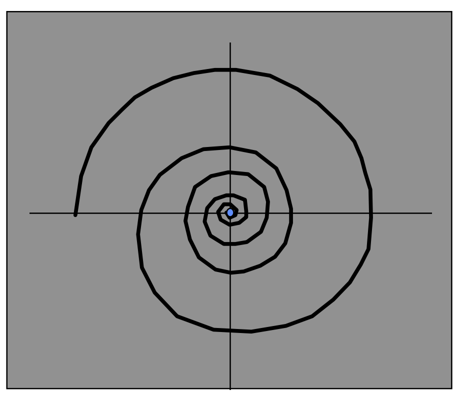

Figure 10. The Attractor of a Simple Pendulum

Now that we've explored some of the most fundamental concepts of Chaos, let's see what they mean in terms of predictability and control.

Putting It All Together

At the risk of beating a horse that's dead, buried, and decomposing; Chaos is not randomness. Nor is it periodicity. Having looked at some basic concepts of Chaos, we are now in a position to see why those differences are important.

To begin, let's first look at a random system (Figure 11). The left figure represents "raw" data as it changes with time. The right figure shows this data in phase-space. Notice how 'jumbled' the data is in phase-space. This is because random events have equal probability of being in any state they *can* be in, from one moment to the next, *independent* of the previous state. For example, consider the classic illustration of random behavior: the coin toss. We know that a tossed coin has only two states available to it: 'heads' or 'tails'. If we toss the coin a million times, half the time the coin will land on heads and half on tails, but we cannot predict which toss will give which state. The 200,550th toss has nothing to do with the first toss, the tenth toss, or even the 200,449th toss. All random events behave in this unpredictable way, each state independent of the next. A random system (Figure 11) has no structure in phase-space.

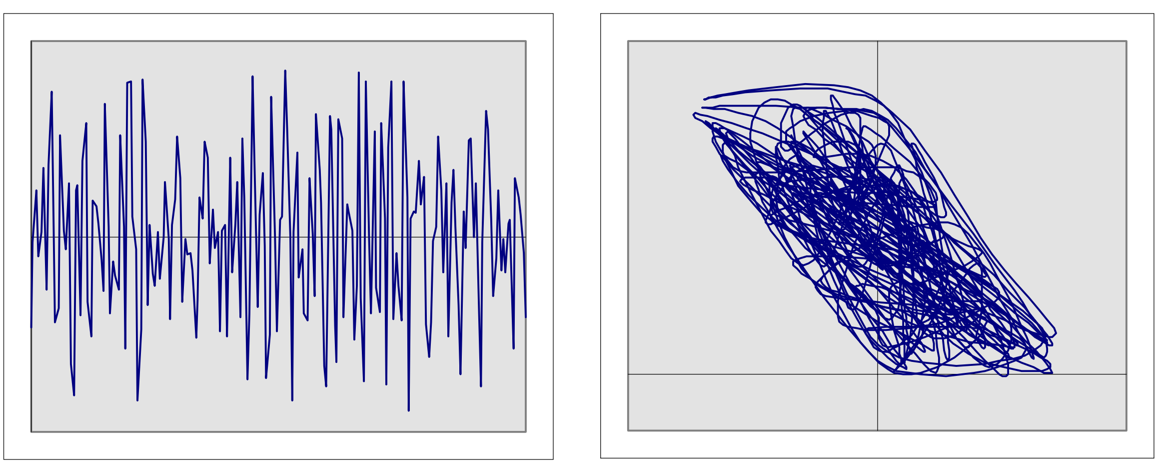

Figure 11. Random Data in "Normal" Space (Left) and Phase-Space (Right).

Now look at the Chaotic data (Figure 12). Again, “raw” data (the output as it changes in time) is shown on the left, and the corresponding phase-space plot of the data is shown on the right. Immediately we see what Edward Lorenz called an ‘orderly disorder‘ in “normal” space (left). It is hauntingly close to having a pattern, and yet still seems so random. But viewing this data in phase-space reveals a decided *structure*: what looks like a canted figure-8. Within this structure the trajectories flow smoothly rather than disjointedly as they do in the random case. In light of our earlier discussion of attractors, the reader can see that there appear to be two attractors in this figure (as opposed to the single point attractors in the periodic data of Figure 9 and Figure 10). Here, the trajectory seems to be ‘drawn’ to, or attracted to orbiting around two distinct lobes, one to the left of center and one to the right.

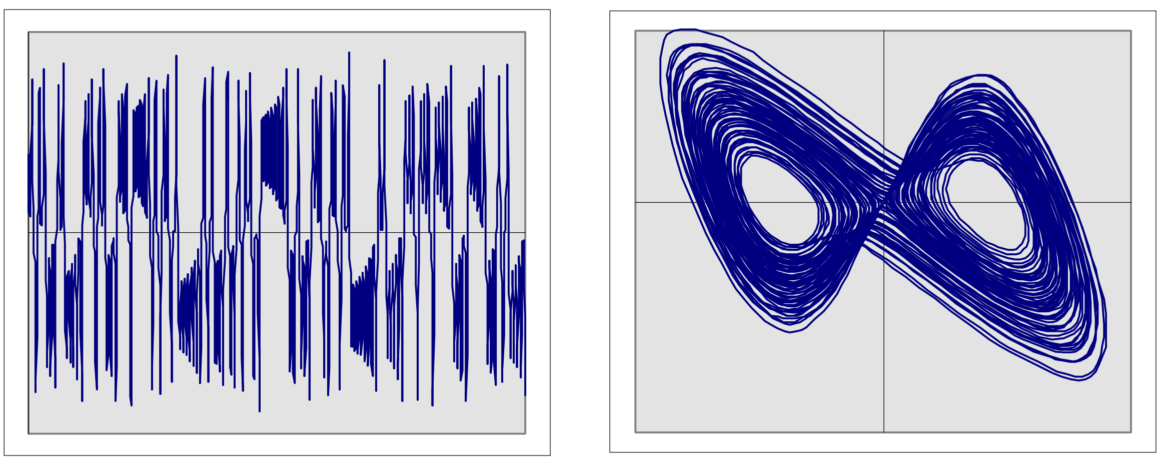

Figure 12. Chaotic Data in "Normal" Space (Left) and Phase-Space (Right).

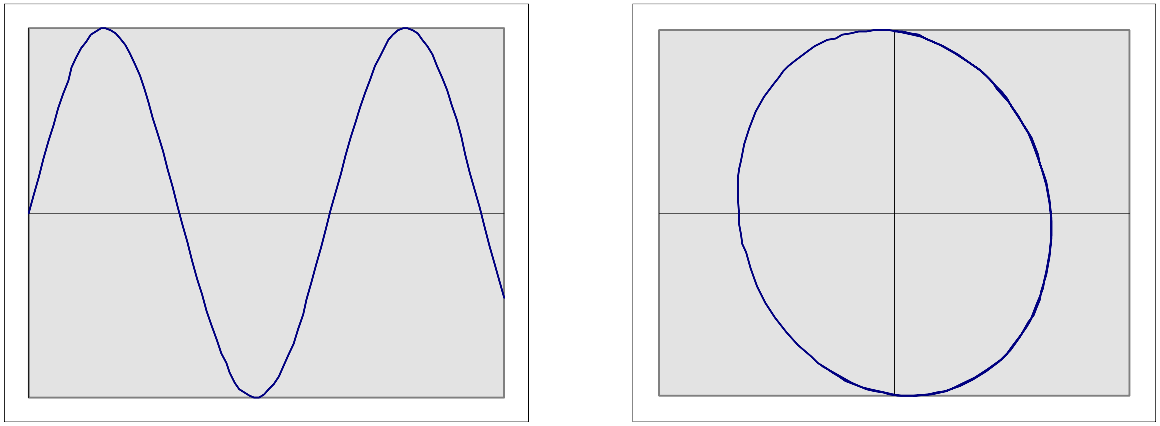

Figure 13. Sinusoidal Data in "Normal" Space (Left) and Phase-Space (Right).

In our example, the attractor of this Chaotic system is dual, it is called a *strange attractor*. A fifth truth of Chaos is that *Chaotic systems **often** have the multi-foci attractors in phase-space called strange attractors.* (Many strange attractors are far more exotic than what is shown in Figure 12. Many are *fractal* in nature. And although a discussion of the fractal nature of Chaos is beyond the scope of this paper, a brief primer on fractals and Chaos is given in Appendix B.)

Going back to Figure 12 and the oddly structured Chaotic data, it is important to note that viewing data in phase-space is the first, often best tool Chaologists have for determining random data from Chaotic data. *The importance of this phenomenon cannot be underestimated*. The very fact that Chaotic data has structure in phase-space, and random data does not, is *why we can make some predictions about Chaotic data*, where we can make virtually none (short of statistical estimates) about random data. Let's develop the logic of this statement.

Consider Figure 14. Here we see how the phase-space plot from Figure 12 is formed with the progression in time. Notice that the trajectory is continuous and smooth, not like the jumbled mess we see with random data. That's because, as said earlier, for random systems each state is independent of the last and so the system evolves in a disjointed manner. However, Chaos is *deterministic*—each state *does* depend on the state before it so the trajectory has a continuity, a sort of flow to it as it evolves in phase-space. B*ut*, as we now know, Chaotic systems are nonlinear and so each state can fluctuate wildly from the one before it (*not discontinuously* just 'wildly'). This is what we're seeing as the trajectory in Figure 14 loops from one lobe of the figure-8 to the next. A 'high' in the data corresponds to a loop around the right lobe; a low corresponds to a loop around the left lobe. We can see this especially in the bottom-middle panel of Figure 14. The two peaks in the data correspond to two loops around the right lobe where the one trough shows up in the one loop around the left lobe. Further, since the left and right lobes correspond to lows and highs in the data, there is no long-term predictability as to how many times the trajectory will loop around one lobe before going to the other—just as *there is no long-term predictability in the pattern of the raw data*. This wild meandering around in phase-space is called *mixing*, and it is important to note that, while the trajectory may come very close to a previous state, the orbit never closes as it does in the case of periodic data. This is another way of saying Chaotic data never falls into a repetitive pattern, though sometimes small portions of the data form patterns that are extremely similar to other portions. It is a sixth 'truth' of Chaos that *all trajectories of Chaotic systems exhibit mixing in phase-space.*

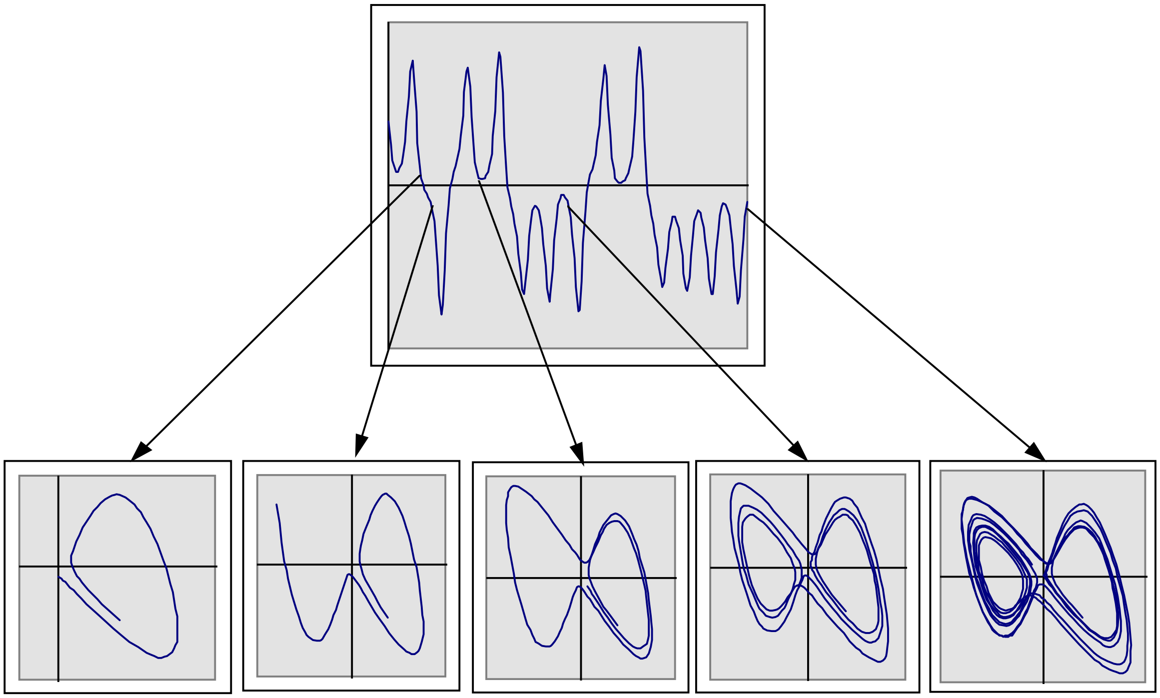

Figure 14. Detailed Evolution of Chaotic Data in Phase-Space

Nonetheless, even though the data swirls around the lobes in an unpredictable way, it still stays within the bounds of the figure-8. Yet another truth of Chaos is that *all Chaotic systems are bounded*, which is exactly what we are seeing when we see the figure-8: the system is constrained within boundaries.

Thus to summarize the previous paragraphs: the deterministic nature of Chaos guarantees a continuity from one state to the next, which is why the trajectory in our example seems to 'flow' in phase-space. But the nonlinear nature means that the changes are extreme, causing, in this example, wild looping from one lobe to the next.

What about another of the 'truths' we discussed earlier, that of sensitivity to initial conditions (SIC)? SIC is also evident in phase-space plots. Recall that as a result of SIC, the slightest variation during the time evolution of the system results in vastly different outcomes. We can see this in Figure 15. The panels in the left-hand column are the same ones from Figure 12, and they correspond to the lighter colored line in the center figure. When we vary initial conditions slightly we get the darker line in the center figure. The frames in the right-hand column are the evolutions of this darker line in phase-space. Notice how, in the center figure, the lines start at about the same point, but very quickly the patterns diverge. We also see this in phase-space. Initially, the phase-space trajectories for both sets of data look very similar (top two frames in both columns). But in the middle frames we see that the original system spends more time orbiting the right lobe, whereas the system subjected to slightly, different initial conditions spends more time orbiting the left lobe. But the *trend* of the data is the same. It still traces out a figure-8 pattern! Thus even though the trajectories wildly mix in phase-space we can still make long-term *trend* predictions! But, because of both the nonlinearity in the system and the tendency for Chaos to be SIC, we *can't* make predictions for more than a few iterations. And, depending how extremely nonlinear and/or SIC the system is, we sometimes can't even do that.

Despite these limitations, hopefully the reader can see from our little graphical foray that there is a definite difference between Chaos, randomness, and periodicity. Bottom line? Chaos is less predictable than periodicity, but more predictable than randomness. It is, as we alluded to in the first chapter, 'something completely different.'

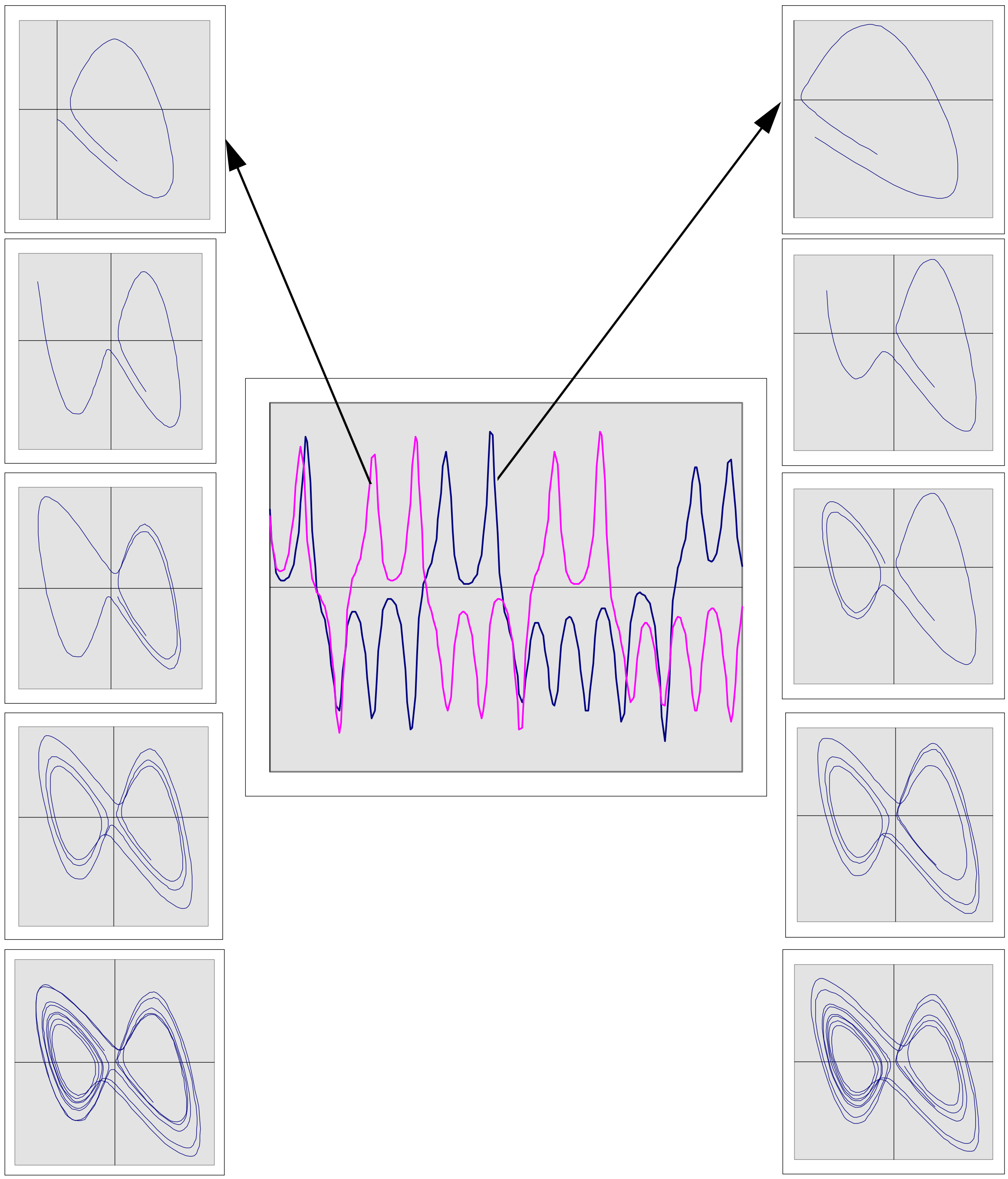

Figure 15. SIC in Phase-Space

Some Practical Advice on Recognition and Control

Recognition

The first step to control is recognition. We've seen in the last section that one of the easiest ways to recognize Chaos is by simply plotting the data as a function of system variables, that is, plotting the data in phase-space. But, in fairness to the reader, this is the easiest, but not the only way, nor is it always such a straightforward procedure.

What was presented in the earlier sections were two-dimensional projections of what can often be multi-dimensional phase-space. For more data with more variables and/or for incomplete data sets, other techniques of analysis need to be used. These techniques include things like calculating Lyapunov exponents, which are a measurement of how "fast" the trajectories represented on a phase-space plot diverge from each other.[6] Other means include calculating quantities with names like Hasdorf dimensions and Capacity dimensions: representations in embedding space, etc., but this is all beyond the level that a vast majority of military professionals need to concern themselves with. It is mentioned here for two reasons. First, so as not to leave the reader with the impression that recognizing Chaos is always simple. The examples used in the paper are completely legitimate, but were chosen for how well they demonstrated the classic nature of Chaotic systems. Second, the author wishes to caution any military professional who may find themselves involved directly with analyzing Chaotic systems, or who work with contractors who will be analyzing such systems, on two points:

1. While there is no "cookbook" for analyzing Chaotic systems, there are a number of software packages developed by credible professionals that describe methods and provide subroutines necessary to perform systematic analyses of Chaotic systems. (One such comprehensive, albeit user-unfriendly, package was used by the author

to generate many of the plots in this paper.)[7] The not-so-subtle point here is *don't pay to develop analysis software* before investigating which Commercial Off-The-Shelf (COT) or Government Off-The-Shelf (GOT) options are available.
2. On the other hand, analyzing Chaotic systems is still more art than science. Although COT and GOT analysis packages exist, they will almost certainly require that whomever uses the packages thoroughly understand them and thoroughly understand Chaos. In other words, *don't use the existing packages like a proverbial 'black-box.'*

The above cautions are the same basic ones military professionals always apply to analyzing military systems. However, because Chaos Theory still has a sort of 'mathematical mystique' around it, these cautions are worthy of repeating here.

Control

Can we control Chaos; prevent its onset, or even induce it? We'll discuss reasons why we might wish to induce Chaos shortly, but the answer to the question can we control Chaos is, *sometimes*, yes.

Recall our example of the logistics equation as it applied to fish populations. One parameter, the average number of offspring per adult, controlled whether that equation produced steady state, periodic, or Chaotic results. But, as above, the author cautions that this simple result should not be too misleading. In this illustrative case the control parameter was obvious. It isn't always that obvious in all cases. That doesn't mean it's impossible. The control parameters for many systems are known, but finding these control parameters is also far more art than science *at the present*. Still, the benefits of being able to drive a system in or out of Chaos *are* obvious. This is an area worthy of the military's attention.

Another means of controlling Chaos, besides finding and 'tweaking' control parameters, is through the introduction of transient phenomenon. Transient motion can be described as "motion that has not yet settled down to a steady and regular pattern."[8] Allan McRobie and Michael Thompson in *Exploring Chaos*, give an example of how several large, nonperiodic waves impacting the hull of a ship can cause the ship to capsize.[9] The author has faith that such considerations are currently part of our Navy's ship building techniques, however, military specialists who are involved with Nation Assistance may find that other, less technically-advanced nations may find this interesting. We will give other examples of systems where considering transient phenomenon may be important in the next chapter.

Summary

The following list summarizes the basic definition of Chaos that we've given in this chapter. This list is a modified version of James' definition of Chaos (for a more expansive definition see James' tutorial).[10]

1. Chaotic Systems *are*
    - deterministic,
    - nonlinear,
    - sensitive to initial conditions,
    - bounded.
2. Chaotic Systems *are not*
    - random, or
    - periodic.
3. The trajectory of Chaotic data mixes in phase-space
4. A chaotic system usually possess strange attractors, often with fractal dimensions.

Because of the nature of Chaotic systems as stated in the above list:

1. There is an underlying structure to Chaotic systems that sometimes allow us to make predictions about
    - long-term *trend* behavior and,
    - *very short-term* behavior but,
    - prevents us from making long-term predictions overall.
2. Some Chaotic systems can be driven in or out of Chaos; that is, Chaos can occasionally be *controlled*.

These are the basic conclusions we can draw about Chaotic Systems. The following chapter gives some specific examples of Chaotic Systems of military concern.

Notes

[1] We can certainly argue that both societal chaos and the Chaos of physical systems are major constituents of battlefield 'fog.' Such a discussion is outside the scope of this paper, but the interested reader may wish to refer to the works of Kelly and McIntosh as cited in the bibliography for relevant discussions.

[2] Actually any value between 1 and 3 gives the same steady result, only taking greater or less time to reach stability.

[3] DeBlois, Bruce. *Deterministic Philosophical Assumptions in the Application of Chaos Theory to Social Events*, Maxwell AFB, School of Advanced Airpower Studies, not published, 3.

[4] Gleick, James. *Chaos: Making a New Science*, New York, Penguin Books, 1988, 15.

[5] Adapted from Gleick, James. *Chaos: Making a New Science*, New York, Penguin Books, 1988.

[6] James, Glenn. *Chaos Theory: The Essentials for Military Applications,* Newport, R.I., Naval War College, 1995, 44.

[7] Sprott, Julian and Rowlands, George. *Chaos Data Analyzer: The Professional Version*, New York, Physics Academy Software, American Institute of Physics, 1994.

[8] McRobie, Allan and Thompson, Michael. *Exploring Chaos*, ed. Nina Hall, New York and London, W.W. Norton and Company, 1991, 150.

[9] McRobie, Allan and Thompson, Michael. *Exploring Chaos*, ed. Nina Hall, New York and London, W.W. Norton and Company, 1991, 150.

[10] James, Glenn. *Chaos Theory: The Essentials for Military Applications,* Newport, R.I., Naval War College, 1995, 44.

Chapter 3

Applications

Now that science is looking, chaos seems to be everywhere . . . Chaos appears in the behavior of the weather, the behavior of an airplane in flight, the behavior of cars clustering on an expressway, the behavior of oil flowing in underground pipes. No matter what the medium, the behavior obeys the same newly discovered laws. That realization has begun to change the way business executives make decision about insurance, the way astronomers look at the solar system, the way a political theorists talk about the stresses leading to armed conflict.

—James Gleick
Chaos: The Making of a New Science

Hard Science

In the first chapter, the author made the seemingly presumptuous statement that there are countless, real, physical systems *upon which military lives and missions depend that are Chaotic systems*. This section summarizes just a handful of these so that the reader can gain a sense for the breadth of such systems.

In his book, Gleick remarked that "Turbulence was a problem with a pedigree." It is certainly a problem of great importance to the military. Turbulent flow affects a multitude of military systems. Flow over aircraft wings is turbulent. The Shuttle must deal with the turbulence at the entire range of atmospheric conditions and over a wide range of speeds. Paratroopers jumping out of airplanes encounter not only turbulence from the effects of air flowing around the aircraft, but, it has been suggested to the author, they might also be affected by turbulence caused from previous jumpers. Turbulence endemic to the atmosphere itself, can affect imaging from space. One of the Air Force's Space "Battle Labs," the Phillips Laboratory, Kirtland AFB, New Mexico, has worked on the effects of Chaotic turbulence on imaging through the atmosphere for almost two decades. This work is intrinsic to "atmospheric characterization, wave front correction and image processing".[1] Construction of adaptive optics sensors that take into account turbulence through the atmosphere have been one of the most significant contributions to the military's Space Object Identification mission in decades. The effects of turbulence in the atmosphere applies to more than just imaging. It has application to propagation of lasers through the atmosphere, for example.

Beyond atmosphere-related issues, turbulence flow is found in the mixing of fuels in some missiles, in aircraft and virtually any liquid-propelled engine. Turbulence flow is found in water pipes that military engineers build through rugged terrain for temporary basing and in the pipes that feed the water fountain in the Pentagon. Turbulence affects nearly every aspect of the military.

While the propagation of lasers through the atmosphere involves turbulence considerations, lasers themselves are affected by nonlinear fluctuations that appear to be Chaotic.[2] Lasers are becoming decidedly ubiquitous in the military today, used in applications ranging from medical to imaging to weapon building.

Other Chaotic systems of great interest to the military may be found in the human body. Dr. Ary Goldberger of Harvard Medical School has argued that "…Chaos gives the body flexibility to respond to different kinds of stimuli, an in particular that the rhythms of a healthy heart are Chaotic."[3]

We have already made note of how transient wave phenomenon can effect the stability of ships, possibly resulting in capsizing. Other examples of transient phenomenon potentially driving systems of military interest Chaotic include the effects of earthquakes on nuclear power plants, offshore oil platforms buffeted by waves, or even printers undergoing internal vibrations.[4]

Electrical circuits are havens for the development of Chaotic behavior. Almost everyone is familiar with the significance of feedback in audio and electrical systems. Feedback that cannot be compensated for can drive a system into Chaotic behavior and can drive it out of such behavior in electrical systems just as we've seen it can do in the case of the logistics equation. In his article *Chaos on the Circuit Board*,[5] Jim Lesurf states that "(amplifiers) can compare the fed-back signal with the original, and any difference between them can be used to correct the output, predicting a better result.…Feedback can be tremendously helpful in reducing nonlinearities. But it must be applied with care, because adding feedback to a nonlinear circuit with gain is a recipe for chaos."[6] He further goes on to remark that "Military communication systems, for example, sometimes transmit radio signals designed to 'hide' in the background noise. These cannot actually be random, otherwise they would not convey any information, but they should imitate real noise as closely as possible to avoid being noticed by eavesdroppers. The kinds of signals produced by 'chaotic oscillators' may be ideal for this."[7] Lesurf expands on this idea in another nifty little article entitled "A Spy's Guide to Chaos,"[8] in which he suggests ways of using Chaos for codemaking and codebreaking.

Whether as a consequence of our military environment expanding outward into space or in response to the recent popularization of “catastrophic scenarios”, the subject of asteroids is of increasing interest to the military. It is hypothesized that asteroids ‘gaps’ (regions almost totally devoid of asteroid such as the Kirkwood Gap which lies between the orbits of the Earth and Mars), are the result of resonances between these regions and the orbits of the planets—Jupiter in particular. In essence, these regions where objects might orbit, have been “cleared out” by the “gravitational tug” of other planets. These regions are not *completely* empty, however, and these gaps include a few asteroids large enough to cause catastrophic consequences if they impacted the Earth. Calculations have shown that “Chaotic orbits of objects at the 3:1 (Kirkwood Gap) resonances could become eccentric enough for them to start crossing the Earth’s orbit.”[9] In the light of the spectacular impact of Comet Shoemaker-Levy on Jupiter in 1994, today, a small but growing portion of military resources are being allocated to study the potential impact of large asteroids with the Earth. This is going to have to include a consideration of the Chaotic nature of the trajectories of heavenly bodies.

So What?

While this list has not been exhaustive, it has hopefully given the reader a feeling for the broad scope of applications for Chaos Theory. This doesn’t mean that *all* military professionals should rush out and become Chaos experts. Without argument Chaos exists in many physical systems. But the average pilot, for example, doesn’t need to understand the physical causes of turbulence on aircraft wings. He or she only needs the wings to ‘stay on,’ but doesn’t necessarily need to know why they do. Indeed, many particularly practically-minded military professionals might be quite underwhelmed with Chaos Theory at this point. Thus far, this paper has been solely concerned with Chaos in physical systems. Many people don't deal with analyzing these types of systems, so, they might think, why should they care? The reason is—much to the dismay of many Chaologists—even though Chaos Theory is strictly a mathematical theory it is being applied to many non-physical applications. Like it or not, Chaos Theory has found its way into the *philosophy* of warfare, in the same way Clausewitz applied 'good old-fashioned' Newtonian linear thinking when he coined the idea of military 'centers-of-gravity.' Furthermore, it has also become very popular for political scientists and military strategists to apply the tenets of Chaos Theory to such soft-science issues as political science and international relationships. Therefore, even for the military professional not specifically concerned with physical Chaotic systems, he or she might encounter Chaos Theory as applied to non-mathematical systems (or at least encounter attempts to do so). And understanding of what Chaos Theory really *is*—deterministic, non-random, nonlinear, and so on—should give the reader better insight into how viable these attempts are. The next sections briefly outline the positives and pitfalls of applying mathematical theory to non-mathematical phenomenon.

Metaphor

There's nothing wrong with a good metaphor—the military loves metaphors. We teach them at our professional military schools and even incorporate them into our doctrine. For example, when the early nineteenth century military strategist, Carl von Clausewitz, introduced the idea of military 'centers-of-gravity' (COG) he was applying a metaphor of linear, Newtonian logic. In physics, the center of mass (gravity) of a system is that point which moves as if the entire mass of the system were concentrated there and all external forces were acting only on that point. Current doctrine has embraced this metaphor passionately, redefining center of gravity metaphorically, as "Those characteristics capabilities, or localities from which a military force derives its freedom of action, physical strength, or will to fight."[10] This is not a bad redefinition (although a surly physicist could pull it apart in a heartbeat). But the metaphor of the COGs does what it needs to do. It forms a frame of reference based upon a generally well understood concept, thereby facilitating discussion and development of ideas. What more could we want from a metaphor? Most military professionals today, especially as they reach higher ranks and deal more frequently with strategic issues, recognize the importance with keeping up with the philosophy of warfare and all the metaphors we commonly use. Especially since current doctrine *enforces* the need to remain cognizant of *popular* metaphors. For example, Joint Publication 5-0 *requires* that military planners consider enemy and friendly COGs in the creation of campaign planning, among other things.[11]

What does this have to do with Chaos Theory? Simply this: Clausewitz is being updated. Graduates of our professional schools and other military professionals alike are re-exploring Clausewitzian COGs, as well as other metaphors that were based on the kind of 'Pre-Chaos World View' DeBlois presented in Figure 5. Two of the better examples of Clauswitz being rewritten in light of Chaos theory are referenced in the bibliography. They are "Modern Scientific Metaphors of Warfare" by Patrick Kelly III, and "Quality, Clausewitz and Chaos," by Richard McIntosh, with the latter possessing a good summary of Chaos Theory itself. Thus, military professional interested in philosophy and strategy of warfare are quite likely to be seeing Chaos Theory in the future. A working knowledge of Chaos Theory should be quite helpful to those individuals.

Soft Science

One area of growing interest in Chaos research today is whether Chaos Theory—a mathematical description of deterministic instability—is applicable to 'soft science' issues such as economics, politics, or sociology. In the quote beginning this chapter, we saw that Gleick mentions that Chaos Theory has inspired "political theorists talk about the stresses leading to armed conflict." Many authors are now writing about Chaos Theory as it might apply to politico-military situations. Some do so logically and some do not. As an example, Alan Saperstein, is logically and systematically investigating the applicability of the Chaos Theory to topics of military interest ranging from "Cold War" arms-race scenarios to issues of Post-Cold War, world wide and regional political stability.[12] A professor of physics and a member of the executive committee in the Center for Peace and Conflict Studies, Saperstein has had a Foster Fellowship with the U.S. Arms Control and Disarmament Agency and Fellowships with the Fullbright Foundation and the National Science Foundation. He is, obviously, a credible source, and his theories are making an impact on the political and military strategic planning communities. Thus, as with the case of rewriting Clausewitz, military professionals are going to see Chaos Theory applied more frequently in the future to politico-military issues among other such soft science applications.

But, while it is reasonable to apply Chaos on a metaphorical level, is it really feasible to apply what is a mathematical theory to situations driven by human factors, such as those which fall under the regime of so called ‘soft’ sciences? That is to say, does it really make any sense to try to calculate ‘bifurcations in political memberships of treaty organizations,’ or try to identify a hidden structure in the Stock Market using phase-space analysis? The answer to this, in part, depends on whether or not the ‘human factor’ driving such situations—for want of a better term ‘free will’—is deterministic or random. We have said repeatedly that Chaos is deterministic. Consequently, if we believe that free will derives from the same inherently unpredictable stochastic causes as randomness does, we simply can’t apply the mathematical analysis techniques of Chaos to societal situations that are driven by this stochastic free will. DeBlois, for example believes this way, as we’ve seen in Figure 5 and Figure 6 that the author borrowed from his insightful paper.[13] On the other hand, there’s no proof that free will isn’t deterministic. Consider the old joke, “If are brains were simple enough for us to understand, we’d be too simple to understand them.” Those who regard free will as deterministic could argue that thought and free will are nothing more than a manifestation of ‘cosmic programming’; a sort of underlying determinism that is to complex for us to understand (that is, our brains are basically computing machines driven by the same causes that drives the physical universe; we’re just not smart enough to understand how). If this were the case, and free will is deterministic, then perhaps we *can* apply mathematical techniques of Chaos Theory to societal situations. And, if we are able to apply mathematical techniques of Chaos Theory do apply to societal situations, what does that say about how ‘free’ free will is?

But what value are these kinds of arguments to a practical mind? If nothing else, to show that we must be very careful not to blindly apply the mathematical techniques of Chaos Theory to soft science issues. The reader needs to keep in mind that Chaos Theory is a mathematics-based description of how real, physical systems behave. We cannot expect this theory to apply to situations that are driven by random, or similarly stochastic conditions. If human factors such as free will aren't stochastic, we might be able to apply Chaos Theory to societal situations. But, if free will is stochastic, then it might make no sense to apply mathematical predictions based on deterministic Chaos to such systems. While many people may yet think of Chaos Theory as 'fashionable number crunching,' many believe that the real 'smoke and mirrors' may lie in the attempts to apply it to non-mathematical soft-science issues. Yet, as strategic thinkers become more interested in the concept of nonlinear, bounded, deterministic behavior in social situations, the military professional is increasingly likely to encounter attempts to apply Chaos Theory from topics which range from updated Clausewitzian metaphor to the "bifurcation in regional politics." Hopefully, the reader is now better prepared to evaluate and/or discuss such topics.

Notes

[1] McMackin, Lenore; Voelz, David and Fender, Janet, *Chaotic Attractors in the Transition Region of an Air-Jet Flow*, [unpublished, n.d.], 1.

[2] *Chaos Data Analyzer: The Professional Version*, New York, Physics Academy Software, American Institute of Physics, 1994., 43.

[3] *Chaos Data Analyzer: The Professional Version*, New York, Physics Academy Software, American Institute of Physics, 1994, 41.

[4] McRobie, Allan and Thompson, Michael, *Exploring Chaos: A Guide to the New Science of Disorder* ed. Nina Hall, New York and London, W.W. Norton and Company, 1991, 151

[5] Lesurf, Jim, *Exploring Chaos: A Guide to the New Science of Disorder* ed. Nina Hall, New York and London, W.W. Norton and Company, 1991, 164.

[6] Ibid., 173.

[7] Ibid.

[8] Ibid., 29-33.

[9] Murray, Carl, *Exploring Chaos: A Guide to the New Science of Disorder* ed. Nina Hall, New York and London, W.W. Norton and Company, 1991, 103.

Notes

[10] Joint Publication 5-0. “Doctrine for Planning Joint Operations,” (13 April 1995), GL-4

[11] Joint Publication 5-0. “Doctrine for Planning Joint Operations,” (13 April 1995), III-18, paragraph 19.

[12] Saperstein, Alvin, *War and Chaos,* American Scientist, 1992.

[13] DeBlois, Bruce. *Deterministic Philosophical Assumptions in the Application of Chaos Theory to Social Events*, Maxwell AFB, School of Advanced Airpower Studies, not published.

Chapter 4

Thinking ‘Chaotically’

“Why,” said the Dodo, “the best way to explain it is to do it.”

—Lewis Carroll
Alice’s Adventures in Wonderland

Thus far, this paper has focused on definitions. But to really understand a concept one has to use it, or, as Alice’s very practically-minded friend the Dodo espoused, the best way to explain a thing is to do the thing.

How then do we ‘do’ Chaos? How do we learn to apply this new view of reality to the world around us? One obvious way would be to pick a physical system, like a wing subjected to turbulent flow, and work through the issues of finding and controlling Chaos in this system. But first, such a tract would be mathematically outside the scope of this paper, and second, it’s likely to be of little interest to many individuals who aren’t aerospace engineers or the like. Instead, in keeping with the spirit of Chaos being ‘something completely different,’ let us focus on doing something distinct from simply looking at well-known examples. Let us focus on *process* by examining the type of questions that should be asked when analyzing a system for Chaos.

To do this, the author has deliberately chosen an example of a system that, at the time of this writing, has not been systematically studied. The positive aspect to doing this is that we won’t be confined by assumptions and conclusions already made, but instead can make our own. However, this approach means that this section becomes an open-ended discussion, resulting (hopefully!) in more questions than answers. This tact might be disturbing to some. Further, this is one person's (the author's) suggested way of thinking about the issues, and thus the conclusions are a matter of the author's judgment. This section is not meant to represent the one and only means of working through the issues, nor is it even intended to be exhaustive. It is simply the suggested first steps on a journey to learning a new way of thinking about the world around us. Hopefully, most will find it an interesting exercise in learning to 'think Chaotically.'

The first risk we will take in this section is defining a system that is not a cut-and-dry physical system, like a bridge, or engine, or aircraft. Instead, we will look at a system-of-systems: the multiple centers-of-gravity (COGs) and parallel warfare. This is a topic of increasing interest in the wake of Desert Storm, where the techniques of parallel warfare proved very successful for the Coalition Forces. In this context, parallel warfare means the ability to strike at several centers-of-gravity (COGs) in diverse locations, simultaneously. This is in contrast to the notion of serial warfare which involves one nation's fielded forces engaging another's directly, and attempting to 'roll them back' from the front lines to the interior of the battlefield.

Our 'system' consists of multiple COGs; the input to the system are targeting decisions and actions made by friendly forces; and the output of the system is degradation of enemy capabilities. Following an idea of DeBlois,' this can be expressed as shown in Figure 16. To make this notion more quantifiable, we will use Col (ret) John Warden's *Five-Ring* model to visualize COGs as segmented into five categories, or nested rings: leadership, organic essentials, infrastructure, population, and fielded forces (Figure 17).

This model assigns greater importance to striking at 'inner ring' targets over those in outer rings, the assumption being that a nation is less likely to tolerate and/or be able to absorb the losses of loosing 'inner ring' systems. The model assumes that each ring can also be broken into self-similar rings. For example, a COG within the organic essentials ring might be electrical power production. Within the system of oil production there are elements of leadership, infrastructure, and so on.

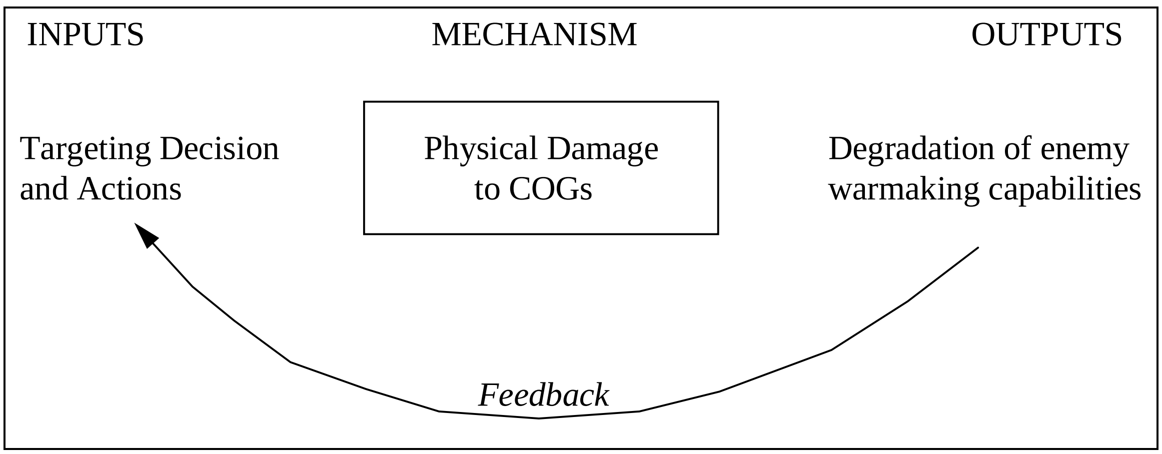

Figure 16. Test Case System

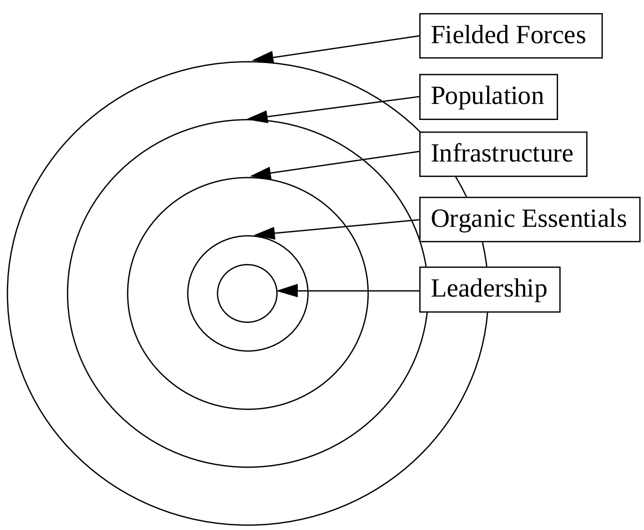

Figure 17. Warden's Five-Ring Model

To investigate the potential use of Chaos Theory for parallel warfare, we need a yardstick; a means to measure the system against, to decide if the principles of Chaos Theory apply (either mathematically, or metaphorically). A reasonable 'yardstick' might be the basic concerns as outlined in this paper in the form of the following:

1. Is the system driven by deterministic or stochastic causes?
2. Is it nonlinear? bounded? sensitive-to-initial conditions?
3. Does the output exhibit mixing or strange attractors?
4. Are there control parameters?

We begin by asking perhaps the most important question first. Is the system driven by deterministic or stochastic processes? Unfortunately, in this case, this is the hardest question to answer unambiguously. At one extreme, we could say that the input certainly doesn't appear to be purely random. The decisions of COGs to attack isn't decided by flipping coins or some other such process (even though the outcome may sometimes look that way to the most cynical among us). Decisions are based on strategic political, military, and economic choices. Further, the output effects the input; that is one state (choice of targets) does depend on the previous one (effect on previous targets). There is feedback. *However,* individual free will and group decision making (which has a 'will' of its own) greatly influences the procedure. Those who do not believe that free will is stochastic might not be bothered by this and might conclude that this system certainly appears to be deterministic. But those who believe free will is strictly stochastic, might have a great deal of difficulty with believing that any system that involves free will as input can result in *quantifiable* Chaos. They may believe that it isn't logical apply the mathematical notions of Chaos Theory to any situation involving human decision making. For those individuals the answer to whether Chaos Theory applies to any type of warfare ends here; it does not. However, we might argue that there is a difference between a system *driven* by free will and one that is *influenced* by free will, much like there is a difference between driving forces and transient forces.

So, is our test system deterministic or stochastic? The above argument is not intended to answer that unambiguously, but instead to engage the reader reaching his or her own conclusions.

For the sake of argument, let's continue to measure this system against some of the other tenets of Chaos. For example, does the system exhibit nonlinearity? One might argue that the very notion that some COGs are thought to create substantially greater degradation to an enemy's capabilities than others (that is, COGs from the inner rings are more important than from the outer rings) might be an example of nonlinearity. But it's not easy to quantify how much greater, thus it's hard to say if that is an example, even metaphorically, of nonlinearity.

What about some other potential nonlinearities in the system? Let's go back to our organic essentials example of electrical production. Sub-ring elements of this COG (which could also be called nodes) are such things as its leadership, workers, power stations, transformer stations, water coming into the system, and power lines moving the electricity out. Suppose, for example the system has two transformer stations, one which feeds a power station and one which feeds five power stations. If we destroy or degrade the first transformer station, we affect one power station, and if we hit the second we destroy five. Is this nonlinearity? Further, each of the degradation of the power stations might affect electrical power in the villages where the workers eat, sleep, and live, thus degrading their effectiveness. Subsequent power outages could also affect the transportation systems that bring the repair crews in to repair the power stations in the first place. Thus, the interconnectedness seems not only to represent a type of nonlinearity, but also is a form of feedback into the system. *At least* on a metaphorical level there appears to be nonlinearity in this system. An analysis of repair rates and effects on production might show more concrete, quantifiable results.

What about boundedness? First, we could quantify boundedness of the system in terms of damage. Doing so would be limited by the accuracy of battle damage assessments (BDA), and would also be subject to definitions, such as: how much destruction constitutes degradation versus destruction or how does repair rates/times figure into BDA. For example, destroying one transformer station isn't likely to destroy the enemy's ability to wage war completely, but anything friendly forces do will have some effect. These are extremely broad definitions, and to be more precise we would have to actually analyze BDA values. The reader might ask themselves at this point, what other examples of boundedness exist in parallel warfare.

What about SIC? One example might be found in going back to the damaged transformer stations. Once the transformers are damaged, crews will rush to repair them. How quickly the repair crews can get their job done will depend on a number of conditions. Daylight versus nighttime, dry season versus monsoon, experienced versus novice crew-persons could make a vast difference in repair rates. In fact, to think of the applicability of SIC to warfare in total one needs only be reminded of Emerson's words:

> For want of a nail, the shoe was lost;
> For want of a shoe, the horse was lost;
> For want of a horse, the rider was lost;
> For want of a rider, the battle was lost;
> For want of battle, the kingdom was lost!

Sensitivity to initial conditions certainly seems to apply. Further, the issue of non-periodicity seems quite clear cut. Repeating the same input, will almost certainly never give the same output.

As for questions about the behavior of the system in phase-space (mixing, strange-attractors), unfortunately, there is no data present from which to derive this information. But this does lead us to the question, what variables might we be looking at is we had the data. This is an important question because it addresses the issue of control parameters. For example, some logical control parameters of this system would seem to be repair rates and frequency of re-strikes. Suppose we could graph the BDA of multiple COGs as a function of how quickly the targets could be repaired versus how frequently we needed to re-attack the targets to ensure a desired level of degradation. Would these parameters be enough to find structure that we could exploit? Could we discover, for example, a pattern of re-strike tempo that might help us maximize damage and minimize wasted resources. Or is it all 'a load of fashionable number crunching?'

Unfortunately, without data, the answer as to whether or not our system is Chaotic from a purely mathematical viewpoint isn't possible at the moment. There *is* reason for regarding it metaphorically as Chaotic. But as stated in the beginning, this section wasn't intended to unambiguously answer all questions. The system presented was simply one the author thought might be worth further investigation if the data can be found. Instead, our purpose here was to present a *process* for looking at systems from a Chaos point of view. If nothing else, hopefully, the reader now understands that *except* in the case of physical or mathematical systems, applying the definition of Chaos to societal situations is often a matter of assumptions, judgments and opinions.

Chapter 5

Conclusions

> *The peaceful partnership society of our Cro-Magnon ancestors was characterized by the three-way cooperation of three principles, Chaos, Gaia and Eros, symbolized by the triple-headed goddess TriVia, the Pythagorean Y, and the victory symbol that emerged during World War II, and in the peace movement of the 1960s. The healing of our planetary society from scars of the past six millennia, the Periodic Epoch, will be an Orphic enterprise. We must welcome the Chaotic Epoch.*
>
> —Abraham
> *Chaos, Gaia, Eros*

The above quote, sadly, is not a joke (unless the entire book is an inside joke). Instead, it is an example of the increasing number of books that portray Chaos Theory as some Grand Unifying Metaphor (GUM) for everything. The author is not attempting to denigrate the work of Abraham, but this quote does show why Chaos is considered by many to be ‘a load of fashionable number crunching’ at best, or a kind of theory du jour of cocktail party scientists, at worse. Unfortunately, there is a vast swell of books on Chaos Theory that give nothing more than lip service to the real underlying mathematical principles of Chaos. The author hopes that this paper has helped the reader understand these principles, at a conceptual level, so as to be able to judge for himself of herself when Chaos Theory applies and when it doesn’t. Those issues the author considers the most important for the reader to remember at the conceptual level are:

Chaos is deterministic, nonlinear, non-random, and non-periodic;

1. It is a different way of viewing reality from the previous ideal that systems could only be completely unpredictable and random, *or* completely predictable;
2. Because Chaos is not random, we can often make long term trend predictions about behavior, and occasionally make very short term predictions as well;
3. Chaotic systems are ubiquitous and military lives and military missions may depend on knowing the difference between Chaos and randomness, and on knowing when a previously well-behaved system may suddenly become Chaotic;
4. Military philosophers and strategists are likely to see a rise in the application of Chaos Theory to metaphorical and soft-science issues;
5. While Chaos can be applied logically at a metaphorical level, there are many questions as to how and *if* Chaos Theory can ever be applied in the soft-sciences.

Chaos is truly a different way of looking at the world around us. For some, this will be a profound difference. For others it won't. Hopefully, this paper has provided the reader with enough information for him or her to decide how important Chaos Theory is to his or her world view.

Appendix A

The Logistics Equation

The logistics equation is a canonical example of Chaos. Used largely in population biology studies, it shows how the population of a species can vary year, by year, based only on the previous year's population and the average number of offspring per adult.

As an example, assume the logistics equation is used to predict fish repopulation. The actual population, given between a value of 0 and 1, at any future time t+1 is designated xt+1. This value simply depends on the last value of the population, xt, multiplied by the number of offspring that each adult produces in the absence of overcrowding, λ, multiplied by a factor that represents the "feedback from effects due to density or crowding[1]", $(1-x_t)$. Thus the logistics equation is:

$$x_t = \lambda x_t(1-x_{t+1}) = \lambda x_t - \lambda x_t^2$$

Equation 1.

Note that this is a nonlinear equation, which, in its most basic definition means that the output of the equation is not directly or inversely proportional to its input. In this equation the output is proportional to the square of the input (see Glossary definition of nonlinear). The modern physics education which the author was subject to, shies away from comprehensive treatment of nonlinear equations—except for the rare few that can be solved with convenient "tricks of the trade" such as perturbation theory—for very good reasons. Although dynamical (i.e., time-dependent) nonlinear equations can be very simple, as is the logistics equation, they can produce “nasty” results, such as a sudden loss of stability for no obvious reason. Again, using the logistics equation as an example we note that if we put set λ (the number of offspring each adult produces) between 1 and 3, the population rises to a “saturation” level (see Figure 1). But if each adult produces more than three offspring a bifurcation (Figure 2) occurs. (Bifurcation is another word that people seem to love to misuse. A bifurcation refers to drastic change in the dynamical pattern, i.e., the quantitative state of a system—see the Glossary). What’s happening is that the population overshoots a stable solution (i.e., “booms”), so the feedback induced by the $(1\text{-}x_t)$ term causes the a “bust,” the next year, for which the feedback again overcompensates, causing another boom and so on and so on. Hence the saw tooth pattern which, in this case represents what is called, period doubling, or period-2. Now if we increase λ again, to about 3.6, the nonlinear feedback induces even more dynamical change; the period doubles again to a period-4 response Figure 3). It’s all a function of feedback. But as anyone who has ever set up their own audio system knows, feedback can be a nasty thing. If the feedback is too large for the system to “compensate” for the system goes unstable; it goes Chaotic! Figure 4 shows this “apparently random” pattern. This behavior is a function of the nonlinearity of the equation. In fact, all Chaotic systems are nonlinear. Further, linear systems can’t be chaotic.[2]

Notes

[1] May, Robert. *Exploring Chaos*, ed. Nina Hall, New York and London, W.W. Norton and Company, 1991, 83.

[2] James, Glenn. *Chaos Theory: The Essentials for Military Applications,* Newport, R.I., Naval War College, 1995, 42.

Appendix B

Fractals and Strange Attractors

As shown earlier in this paper, many Chaotic systems demonstrate an odd, "multi-focal point" structure in phase-space called a strange attractor, for more complicated than the Lorenz attractor shown in Figure 12. Some strange attractors are fractal. Even though a full discussion of fractals and strange attractors are outside the scope of this paper, the following offers a brief primer. The curious reader is encouraged to read the works of Gleick and James, cited in the bibliography, as the next logical step in examining the relationship between fractals and strange attractors.

Fractals are not a new concept to most people. Almost everyone can recognize the elegantly intricate Mandelbrot Set first brought to light by Benoit Mandelbrot and which seemed to grace every science-graduate student's cubicle in the 1960s. More formally, a fractal can be regarded as a measure of the degree of "roughness," "irregularity," or "fractionation" of an object. A point has a fractal dimension of 0, as it has no "dimension." A line is said to have the fractal dimension of 1, a plane has the fractal dimension 2, and a three dimensional surface has the fractal dimension 3. However a jagged coastline will have a fractal dimension between 1 and 2, a mountain range will have a dimension between 2 and 3. Fractals also possess the quality of being self-similar, that is they maintain the same degree of irregularity at all scales. Fractals are ubiquitous in nature; they are seen in clouds and coastlines, ferns and trees. The human circulatory system is a series of self-similar fractals from "aorta to capillary."[1]

The strange attractors of Chaotic systems are often fractal. This does *not* mean that all fractals are examples of Chaos. Fractal and Chaos are not synonyms, as they often are presented to be in the literature (usually by the same people who use Chaos and random as synonyms).

However, the military might be able to exploit the fractal nature of strange attractors in the area of data compression. Devilishly intricate fractal patterns can be reproduced by a limited set of instructions. Think of the fern plant, which is essentially the pattern of one frond multiplied over and over again. If one could send the pattern of the frond, and a simple set of instructions as to how to reproduce the frond, one could reconstruct the entire plant with less instructions than if one had to send the fern plant pattern in its entirety. As discussed above, coastlines and mountain ridges are fractal. We've also seen that Chaotic systems can be generated by very simple equations. If we could find the equation that represented a coastline as seen from a satellite and transmit only the equation and minimal reconstruction equations we might be able to use less time and bandwidth on our precious transmission links.

Notes

[1] McIntosh, Richard. *Quality, Clausewitz and Chaos: New Science Interpretations of Self-Similar Systems*, Montgomery, Air War College, Air University, 1995, 22.

Glossary

ACSC	Air Command and Staff College
SIC	Sensitive to Initial Conditions
USAF	United States Air Force

attractors. The limit cycles or states that a system settles into after its transient dynamics die out.[1]

bifurcation. The tendency for a system, when one controlling parameter is changed, to drastically change behavior. For example, a system that displays a single period pattern suddenly develops a "beat" pattern. When the period goes from single to a "double-beat" it is called doubling or **period-2**; a four beat pattern is called **period-4**, etc. Bifurcation often precedes the onset of Chaos.

causality. See determinism.

determinism. The belief that the future state of any system will be in can be precisely known if enough is known about the constituents of a system (the components it can be "reduced to") and the conditions effecting that system. Before Chaos Theory is was believed that deterministic behavior always produced steady behavior[2]

fractals. A measure of the degree of "roughness," "irregularity," or "fractionation" of an object. A point has a fractal dimension of 0, as it has no "dimension." A line is said to have the fractal dimension of 1, a plane has the fractal dimension 2, and a three dimensional surface has the fractal dimension 3. However a jagged coastline will have a fractal dimension between 1 and 2, a mountain range will have a dimension between 2 and 3. Fractals also possess the quality of being self-similar, that is they maintain the same degree of irregularity at all scales. Fractals are ubiquitous in nature; they are seen in clouds and coastlines, ferns and trees. The human circulatory system is a series of self-similar fractals from "aorta to capillary."[3]

Lyapunov exponents. A measurement of how "fast" the trajectories represented on a phase-space plot diverge from each other.[4]

logistics equation. A nonlinear equation (see nonlinear) used by biologists to describe the population fluctuations of animal populations. The animal population, **x,** for a future time **t+1**, in (i.e. $\mathbf{x_{t+1}}$) is equal to last value of the population, $\mathbf{x_t}$, multiplied by the number of offspring that each adult produces in the absence of overcrowding, λ, multiplied by a factor that represents the "feedback from effects due to density or crowding[5]", $\mathbf{(1-x_t)}$, to give:

$$x_t = \lambda x_t(1-x_{t+1}) = \lambda x_t - \lambda x_t^2$$

mixing. The complex, interweaving of trajectories (evaluations of states) within phase-space that characterizes Chaotic systems. Though, they may approach the bounds of the limit states of the attractor, and may pass infinitesimally close to other states of the systems, these trajectories never repeat themselves.

nonlinear. In the most basic sense, non-linear means that the output of a system is not directly or inversely proportional to its input. Linear equations contain only addition, subtraction, multiplication or division by constants. Nonlinear operations involve exponents, trigonometric functions and logarithms. All Chaotic systems are nonlinear, but not all nonlinear systems are Chaotic.

non-periodic. Non-repetitive, characterized by never settling into a close loop behavior in phase-space.

phase-space plots. A means of representing the states of a dynamic system by graphing its evolution as a function of the minimum number of time-dependent variables of the system.

reductionism. The practice of analyzing the behavior of an entire system as a product of the behavior of its components.

sensitive to initial conditions. A small change in an initial condition or parameter manifests in a radically different end states.

strange attractors. The complicated, bounded orbits of trajectories of a Chaotic system. Strange attractors possess the property of mixing (see mixing), and are often fractal. It is by virtue of this long-term boundedness that allow limited predictions to be made by some Chaotic systems.

state. The collection of dynamical variables at a given time that describe a system.

trajectory. The time-evolution of the states of a system.

Notes

[1] James, Glenn. *Chaos Theory: The Essentials for Military Applications,* Newport, R.I., Naval War College, 1995, 26.

[2] Gleick, James. *Chaos: Making a New Science*, New York, Penguin Books, 1988, 79.

[3] McIntosh, Richard. *Quality, Clausewitz and Chaos: New Science Interpretations of Self-Similar Systems*, Montgomery, Air War College, Air University, 1995, 22.

[4] James, Glenn. *Chaos Theory: The Essentials for Military Applications,* Newport, R.I., Naval War College, 1995, 44.

[5] May, Robert. *Exploring Chaos*, ed. Nina Hall, New York and London, W.W. Norton and Company, 1991, 83.

Bibliography

Abraham, Ralph. Chaos, Gaia, Eros: A Chaos pioneer Uncovers the Three Great Streams of History, San Francisco, Harper Collins, 1994.

Air Command and Staff College. *Concepts in Airpower for the Campaign Planner*, (Air Command and Staff College, 1993).

Calvin, William. *How Brains Think,* New York, Basic Books, 1996.

Cohen, Jack and Stewart, Ian. The Collapse of Chaos (Chapter 12: The Behavior of Interactive Systems), New York, Penguin Books, 1994

DeBlois, Bruce. *Deterministic Philosophical Assumptions in the Application of Chaos Theory to Social Events*, Maxwell AFB, School of Advanced Airpower Studies, not published

James, Glenn. Chaos Theory: The Essentials for Military Applications, Newport, R.I., Naval War College, 1995.

Joint Publication 5-0. "Doctrine for Planning Joint Operations." (13 April 1995).

Gleick, James. *Chaos: Making a New Science*, New York, Penguin Books, 1988.

Kelly, Patrick III. Modern Scientific Metaphors of Warfare: Updating the Doctrinal Paradigm, Fort Leavenworth, Kansas, 1993.

Mann, Steven. *Chaos, Criticality, and Strategic Thought*, Montgomery, Strategic Operations Coursebook, Air Command and Staff College, 1995.

McIntosh, Richard. *Quality, Clausewitz and Chaos: New Science Interpretations of Self-Similar Systems*, Montgomery, Air War College, Air University, 1995.

Nicholls, David; Tagarev, Tudor; and Axup, Peter, *What Does Chaos Theory Mean for Warfare?,* Montgomery, Air Command and Staff College, Air University, 1995.

Richards, Diana. A Chaotic Model of Power Concentration in the International System, International Studies Quarterly, 1993.

Saperstein, Alvin, War and Chaos, *American Scientist*, 1992.

Saperstein, Alvin, Chaos: A Model for the Outbreak of War, *Nature*, 1984.

Saperstein, Alvin, The "Long Peace," *Journal of Conflict Resolution*, 1991.

Stoner, John. Energizing the Trinity: Operational Implications of Warfare in the Age of Information Technology, Fort Leavenworth, School of Advanced Military Studies, 1993.

Tagerev, Todor, Michael Dolgov, David Nicholls, Randal C. Frankin and Peter Axup. *Chaos in War: Is It Present and What Does It Mean?,* Air Command and Staff College, Maxwell Air Force Base, Alabama, 1994.

Waldrop, M. Mitchell. Complexity: The Emerging Science at the Edge of Order and Chaos, New York, Touchstone Books, 1992.

Warden, John III, *The Air Campaign*, New York, Pergamon Press, 1989.